Two planes on the same route are:
--separated
--trailing plane is traveling faster than the leading plane

SMART SKIES

Airspace Systems—Predicting Air Traffic Conflicts

Teacher Guide

Curriculum Supplement 7

Overview of Curriculum Supplement 7

You may choose to spread the experiment and calculation activities over two or three class periods, allowing time for setting up the experiment, conducting the experiment, doing the calculations, and discussing the outcomes.

This is the seventh in a series of Airspace Systems Curriculum Supplements that address distance-rate-and time problems. Each Curriculum Supplement consists of an experiment, worksheets to support the experiment, worksheets for paper-and-pencil calculations, a student analysis of the airspace scenario, and optional pre- and post-assessment instruments.

In this Curriculum Supplement, the controller must determine when two planes traveling on the same route will conflict with one another. This will determine when the controller will have to take action to keep the planes separated.

Initially, the planes are separated from one another on route. Each plane is traveling at a different constant (fixed) speed. The trailing plane is traveling faster than the leading plane.

This scenario is different from the previous six cases which each involve merging aircraft.

Airspace Scenario

Students will determine when two airplanes traveling on the same route will conflict with one another.

Initially, the airplanes are **separated on route.**
The airplanes are traveling at **different constant (fixed) speeds**.
The trailing airplane is traveling more rapidly than the leading airplane.

Of the eight *Airspace Systems* Curriculum Supplements, this is the first scenario to address two planes traveling on the same route. The current Supplement asks students to determine when the trailing plane will catch up with the leading plane. The previous six Supplements asked student to determine when two planes on two different merging routes would arrive at the intersection of the routes.

As in Curriculum Supplement 6, both the speeds and the distances traveled differ for each plane.

It is highly recommended that your student complete the previous Supplements before they attempt the current Supplement.

Objectives

Students will determine the following:

> If two planes are traveling at different speeds on the same route and the trailing plane is traveling faster than the leading plane, the trailing plane will close the gap at a rate equal to the difference in the speeds of the planes.
>
> So if the difference in speeds is twice as great and the starting distance between the planes remains the same as the original starting separation distance, then the trailing plane will close the gap at twice the original rate. Therefore, the amount of time for the trailing plane to catch up to the leading plane will be half as great.
>
> If the planes each travel at their original speeds but the starting distance between the planes is twice as great, then the trailing plane must close twice the distance at the original rate. So the amount of time will double for the trailing plane to catch up to the leading plane.

Introducing Your Students to the Airspace Scenario

If you have not already done so, you may want to show the "Gate to Gate" CD-ROM to introduce your students to the air traffic control system. (For more detail, see the Smart Skies Airspace Systems Introduction for Teachers.)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, live on the same street. They each leave their homes at the same time and walk in the same direction along their street at different constant (fixed) speeds. Alex starts out 10 blocks behind Ana, but Alex walks twice as fast as Ana. Both students are walking to a school at the end of their street, 30 blocks beyond Ana's house.

You can ask your students this question:

Will Alex catch up with Ana before she reaches the school?

Activity 7.0 --

Problem Statement

In a real-world scenario,

Problem Statement

Worksheet 7.0 describes and illustrates the airplane scenario. The speed of one airplane is 1/2 foot/second. The speed of the other airplane is 1/4 foot/second. The faster plane starts 10 feet behind the slower plane.

one plane's speed might be 400 nautical miles per hour and the other plane's speed might be 320 nautical miles per hour. The initial separation distance might be 10 nautical miles.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

As a problem extension, you may want to ask your students to solve the problem using real-world data.

Note: These speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

One question is posed:

Q1: How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with Flight NAL63?

Student Handout:
Worksheet 7.0

Students may have an intuitive feeling about the results being related to the relative speed of the two planes.

Students can calculate or graph to determine the number of seconds for the trailing plane to catch up with the leading plane.

Materials
Worksheet 7.0: Problem Statement

Activity 7.1 --

Pretest

Estimated time:
15 - 30 minutes

*The pretest is **optional.***

Pretest—Make a Prediction
The pretest steps the student through a careful reading of the airplane problem statement. The student is then asked to predict the outcome of the given airplane scenario.

The pretest may be assigned as either an individual or a small-group activity.

Instead of distributing the pretest, you may want to use the questions to guide a classroom discussion.

Student Handout:
Worksheet 7.1A
Worksheet 7.1B

If your students have completed other Airspace Systems Curriculum Supplements, you may want to direct them to use a particular calculation method or methods to answer the pretest questions. In that case, Worksheet 7.1B contains blank vertical line plots as well as grids that students can use as they do their calculations.

Materials
Worksheet 7.1A: Pretest—Make A Prediction
Worksheet 7.1B: Lines and Grids

Activity 7.2 --

Experimentation

Estimated time:
1 hour after setup

*For a step-by-step student orientation to the Experiment, see Curriculum Supplement 0, the introduction to **Airspace Systems**.*

Student Handouts:
Worksheet 7.2A
Worksheet 7.2B
Worksheet 7.2C

You may want to give students an overview of the experiment including an explanation of what they will do in each activity.

You may want to ask your students to compare the experiment distances and speeds with the real-world speeds given in the sidenote for Activity 7.0.

You may want to ask your students to estimate the route layout before they measure.

Students who have little experience in measurement may benefit from first practicing skip counting (by 6 and by 3) to prepare them to measure 6-inch lengths and 3-inch lengths.

Classroom Experiment

In this small-group activity, students mark off the jet route on the classroom floor or on an outdoor area. Students assume the roles of pilots, air traffic controllers, and NASA scientists. The pilots step down the jet route at a prescribed pace. The NASA scientists track and record the pilots' times and the pilots' distances from the start of the jet route. The air traffic controllers set the pace.

Materials

Activity 7.2A: Set Up the Experiment
- --sidewalk chalk or masking tape
- --measuring tape or ruler
- --marking pens (optional)

Activity 7.2B: Conduct the Experiment
- --1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- --pencils and Data Sheets (Worksheet 7.2C)
- --signs identifying pilots, controllers, and NASA scientists
  Note: the signs are available on the Smart Skies website.
- --clipboard (optional)

Student Handouts:
--Worksheet 7.2A: Set Up the Experiment
--Worksheet 7.2B: Conduct the Experiment
--Worksheet 7.2C: Data Sheet

Worksheet 7.2A, Set Up the Experiment

One pilot will walk down each side of the jet route. Allow enough distance on either side of the route so that the two pilots are not distracted by one another.

You may want to set up one jet route as a model that your students can copy.

After a group of students has completed its jet route set-up, you may find it helpful to have them compare their work with another student set-up.

Worksheet 7.2B, Conduct the Experiment

Assign students to positions on 6-8 person teams as follows:

--Lead Air Traffic Controller (1 student)
--Secondary Air Traffic Controller (1 student)
--Pilots (2 students)
--NASA Scientists, 1 or 2 for each plane (2 – 4 students)

It may be difficult for some student pilots to take 6-inch or 3-inch steps by placing one foot in front of the other. Instead, advise the pilots to place one foot on either side of the jet route and align their toes at each mark. It may be helpful for students to practice.

After the jet routes are set up, have one group of students demonstrate the experiment while the rest of the class observes. Discuss and address any issues that may arise.

Perform the activity at least three times. Compare the results of each trial. Discuss the validity of the results.

Extensions:

1. Repeat the activity using different students as the Air Traffic Controllers, Pilots, and NASA Scientists.

2. Repeat the activity using a jet route longer than 20 feet. Try different starting distances between the planes. Increase the plane speeds and the step sizes to 1 foot/second and 1/2 foot/second, respectively. Use plane speeds in various ratios—e.g., instead of 2 : 1, try 3 : 1.

3. Have students draw a scale model of the experiment using real-world data. (See the sidenote for Activity 7.0).

Activity 7.3 --

Calculations

Estimated time:
15 - 30 minutes per worksheet

Calculate the Time for the Trailing Plane to Catch Up with the Leading Plane

This activity presents six different methods students can use to determine the number of seconds for the trailing plane to catch up with the leading plane.

Each worksheet may be assigned as either an individual or a small-group activity.

You can choose to assign one, some, or all of the worksheets. If students have completed an earlier Curriculum Supplement, you may decide to focus on only one worksheet.

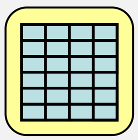
You may want to assign some worksheets before and some worksheets after the experiment.

The calculation methods range in order of difficulty as follows:

- Counting (completing a table)
- Drawing blocks to make a bar graph
- Plotting points on two vertical lines
- Plotting points on a Cartesian coordinate system
- Deriving and using the distance-rate-time formula
- Graphing two linear equations

Worksheet 7.3A, Count Feet and Seconds

Student Handout:
Worksheet 7.3A



Students use patterns and skip-counting to complete a table and solve the problem. At the end of this activity, students may realize it is faster to multiply than to add to obtain the answer.

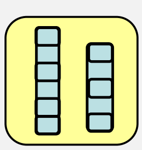
In Curriculum Supplements 1 through 6, the table helped students track the total number of feet traveled by each plane until it reached the end of its individual route (that is, until each plane reached the point where the routes merged). Students could *complete one column at a time for each plane* since the questions were concerned only with each plane's total travel time.

In the current Curriculum Supplement, the table features the number of feet each plane has traveled from its starting point on the same route. So the table shows that the leading plane starts at 10 feet and the trailing plane starts at 0 feet. You may want to suggest that your students first complete the table through the "11 foot row" for Flight WAL27. Then suggest that the students *complete one entire row at a time for both flights*. In that way, at each step, students can look at the data for both planes to see when the planes will be in the same place at the same time.

Prerequisite skills: count by 2s, count by 4s, subtract fractions with different denominators

Worksheet 7.3B, Draw Blocks

Student Handout:
Worksheet 7.3B



Students draw blocks, each representing the distance each plane travels in 10 seconds. The students "stack" their blocks along two vertical number lines (one line for each plane) that represent the jet route.

Notice that the vertical lines are numbered differently from the vertical lines used in Curriculum Supplements 1 through 6.

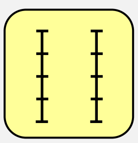
In this Supplement, the lines are numbered from 0 at the *bottom* to 30 at the *top*. Students begin to stack the blocks at the starting point of each plane. The trailing plane starts at 0. The leading plane is 10 feet ahead of the trailing plane and starts at 10.

To help students make the connection between the jet routes and the vertical scales, students are first asked to plot a point on a jet route diagram and then stack the corresponding block along the vertical scale.

Notice that the jet route diagram features six copies of the jet route. These copies represent "snapshots" of the jet route at 10-second intervals. As students plot points on each copy, they will see the trailing plane close in on the leading plane.

Prerequisite skills: read and build a bar graph with a vertical scale marked in 1-foot units; count by 10s, subtract fractions with different denominators

Student Handout:
Worksheet 7.3C



Worksheet 7.3C, Plot Points on Two Vertical Lines

This graph is similar to the way families record and compare the height of their children at the same ages. They mark off each child's birthday height (distance from the floor) on a doorway and then record their age (time since birth) beside the height mark.

The students plot their points along two vertical number lines (one line for each plane) that represent the jet route.

Notice that the vertical lines are numbered differently from the vertical lines used in Curriculum Supplements 1 through 6.

In this Supplement, the lines are numbered from 0 at the *bottom* to 30 at the *top*. Students begin to plot points at the starting point of each plane. The trailing plane starts at 0. The leading plane is 10 feet ahead of the trailing plane and starts at 10.

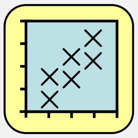
To help students make the connection between the jet routes and the vertical scales, students are first asked to plot a point on a jet route diagram and then plot the corresponding point on the vertical scale.

Notice that the jet route diagram features six copies of the jet route. These copies represent "snapshots" of the jet route at 10-second intervals. As students plot points on each copy, they will see the trailing plane close in on the leading plane.

Prerequisite skills: plot a point on a (vertical) number line, subtract fractions with different denominators

Worksheet 7.3D, Plot Points on a Cartesian Coordinate System

Student Handout:
Worksheet 7.3D



Notice that the vertical axis is numbered differently from the vertical axis used in Curriculum Supplements 1 through 6.

In this Supplement, the vertical axis is numbered from 0 at the *bottom* to 30 at the *top*. Students begin to plot points at the starting point of each plane. The trailing plane starts at 0. The leading plane is 10 feet ahead of the trailing plane and starts at 10.

You may choose to combine this Worksheet with Worksheet 7.3F (Use the Distance-Rate-Time Formula) to help students obtain the distances that correspond to each 10-second time interval.

Notice that the jet route diagram features six copies of the jet route. These copies represent "snapshots" of the jet route at 10-second intervals. As students plot points on each copy, they will see the trailing plane close in on the leading plane.

Note that if the horizontal "tick marks" on each jet route were extended and connected, a grid would be formed. This grid would be equivalent to the grid given at the bottom of the worksheet page.

Prerequisite skills: plot a point on a Cartesian coordinate system (the xy-plane) , subtract fractions with different denominators

Extension (optional):
For each plane, connect the points with a straight line. Find the equation of each line.

Worksheet 7.3E, Derive the Distance-Rate-Time Formula

Student Handout:
Worksheet 7.3E

d = r • t

Students use patterns to derive the distance-rate-time formula in the form d = rt.

Prerequisite skills:
Use patterns to make a generalization.

Worksheet 7.3F, Use the Distance-Rate-Time Formula

Student Handout:
Worksheet 7.3F

Students apply the distance-rate-time formula in the form d = rt.

t = d / r

In Curriculum Supplements 1 through 6, both the distance to the intersection and the rate were known for each airplane. So each plane's travel time to the intersection could be calculated by simply substituting its distance and rate in the formula written in the form t = d/r.

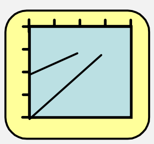
In the current Curriculum Supplement, only the rate is known for each plane. So the formula does not lead directly to the answer. Instead, students are asked to calculate distances for each plane for different times. They examine these results and close in on the time when each plane has traveled the same distance from the starting position of the trailing plane.

You may choose to combine this Worksheet with Worksheet 7.3D (Plot Points on a Cartesian Coordinate System) to help students visualize the relationship between the time and the distances.

Prerequisite skills:
Substitute numbers into a formula.

Worksheet 7.3G, Graph Two Linear Equations

Student Handout:
Worksheet 7.3G



For each airplane, students graph a given linear equation that describes the plane's distance traveled as a function of time.

The point at which the two lines cross represents the collision of the planes. That is, the planes are in the same place at the same time. However, in Curriculum Supplement 6, the common point did *not* represent a collision. In that Supplement, the planes were on *different* routes and the point where the lines crossed merely indicated that at a particular time, each plane was the same distance from the intersection of the routes.

Caution: Students may confuse the path of a plane with the graph of the plane's distance from the intersection as a function of time.

Prerequisite skills:
Graph a linear equation by making a table of ordered pairs.
Find the slope of a line given the equation of the line and the graph of the line.

Extension (optional):
You may want to ask your students to find the y-intercept of each line and interpret those intercepts in the context of the airspace problem.

For each plane, the y-intercept represents the plane's initial distance from the starting point of the trailing plane at time x = 0.

Also note that for each time, x, the vertical distance between the lines represents the distance between the planes after x seconds.

Materials

Worksheet 7.3A: Calculate the time—count feet & seconds
Worksheet 7.3B: Calculate the time—draw blocks
Worksheet 7.3C: Calculate the time—plot on two vertical scales
Worksheet 7.3D: Calculate the time—plot points on a Cartesian coordinate system
Worksheet 7.3E: Derive the Distance-rate-time formula
Worksheet 7.3F: Use the Distance-rate-time formula
Worksheet 7.3G: Graph Two Linear Equations

Activity 7.4 --

Analysis

Estimated time:
45 minutes

Student Handout:
Worksheet 7.4

Compare the Experimental Results with the Predicted Results

Students compare the outcome of the experiment with their pretest predictions.

This activity may be assigned as either an individual or a small-group activity.

If you assigned some calculation worksheets (Activity 7.3) prior to the experiment, students can compare their calculations with the experimental results.

You may want to assign some Activity 7.3 calculation worksheets after the experiment to give students another basis for comparison. As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a school. (See the Airspace Scenario section of this document.)

Now, you might suggest they consider two cars traveling in the same lane on the same road. The cars are each traveling at a different constant (fixed) speed. The trailing car is traveling faster than the leading car.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Materials

Worksheet 7.4: After the Experiment

Activity 7.5 --

Posttest

Estimated time:
15 - 30 minutes

*The posttest is **optional.***

Student Handouts:
Worksheet 7.5
Worksheet 7.1B

Curriculum Supplement Posttest

This activity may be assigned as either an individual or a small-group activity.

You can direct your students to use a particular calculation method or methods to answer the posttest questions. Worksheet 7.1B (used for the Pretest) contains blank vertical line plots and grids that students can use as they do their calculations.

Materials
Worksheet 7.5: Posttest
Worksheet 7.1B: Lines and Grids

Name

Problem Statement

In the picture below, two airplanes are flying on the same route.

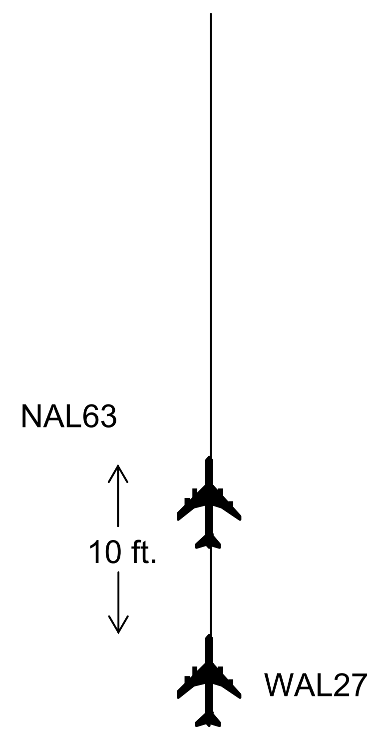
The World Airlines plane has flight number **WAL**27.

The speed of Flight WAL27 is 1/2 foot/second (0.15 meters/second).

The National Airlines plane has flight number **NAL**63.

The speed of Flight NAL63 is 1/4 foot/second (0.08 meters/second).

At the start of the problem, Flight WAL27 is 10 feet (3.0 meters) behind Flight NAL63.



Question: How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with Flight NAL63?

____
Name

Pretest—Make a Prediction

In the picture on the next page, two airplanes are flying on the same route.

The World Airlines plane has flight number WAL27.
The speed of Flight WAL27 is 1/2 foot/second.

1. Write the speed of Flight WAL27 next to its picture.

2. How far does Flight WAL27 travel in one second? ________________

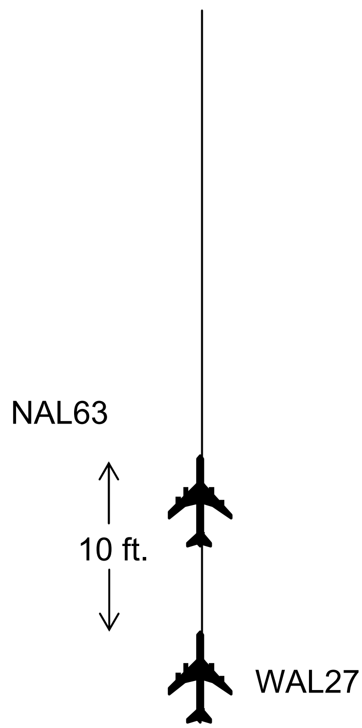
The National Airlines plane has flight number NAL63.
The speed of Flight NAL63 is 1/4 foot/second.

3. Write the speed of Flight NAL63 next to its picture.

4. How far does Flight NAL63 travel in one second? ________________

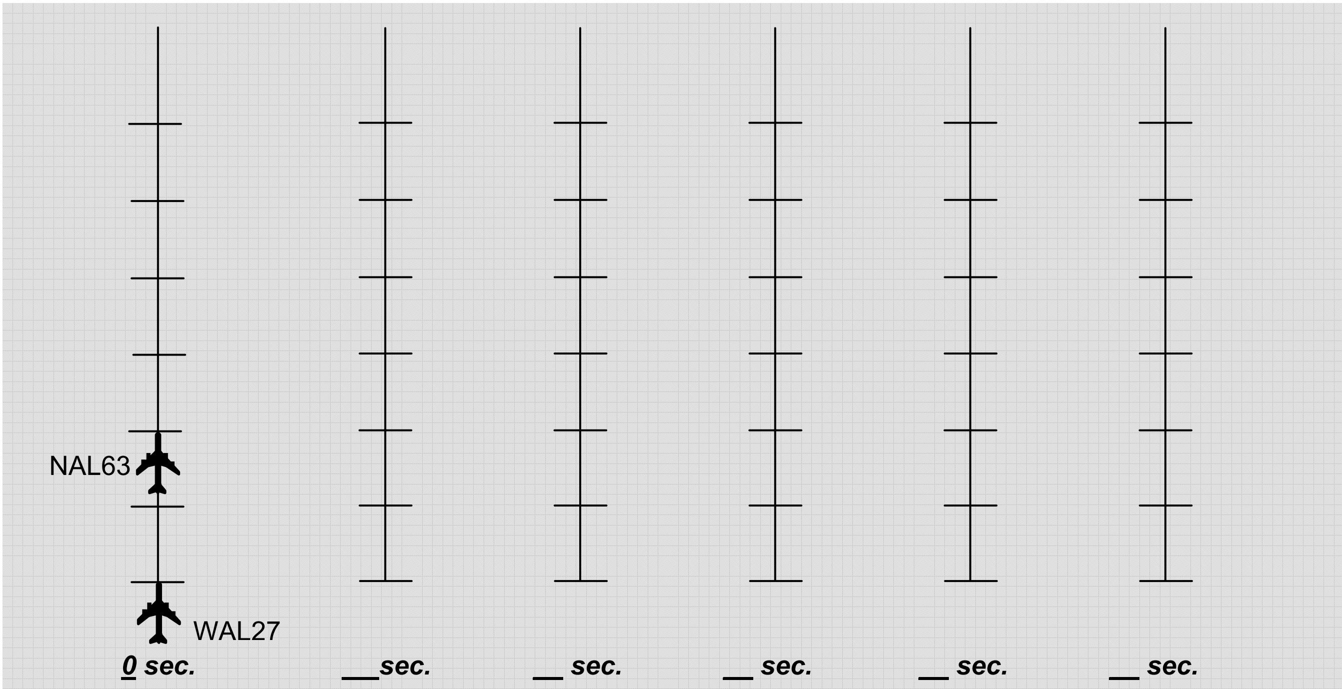
5. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with Flight NAL63? ________________

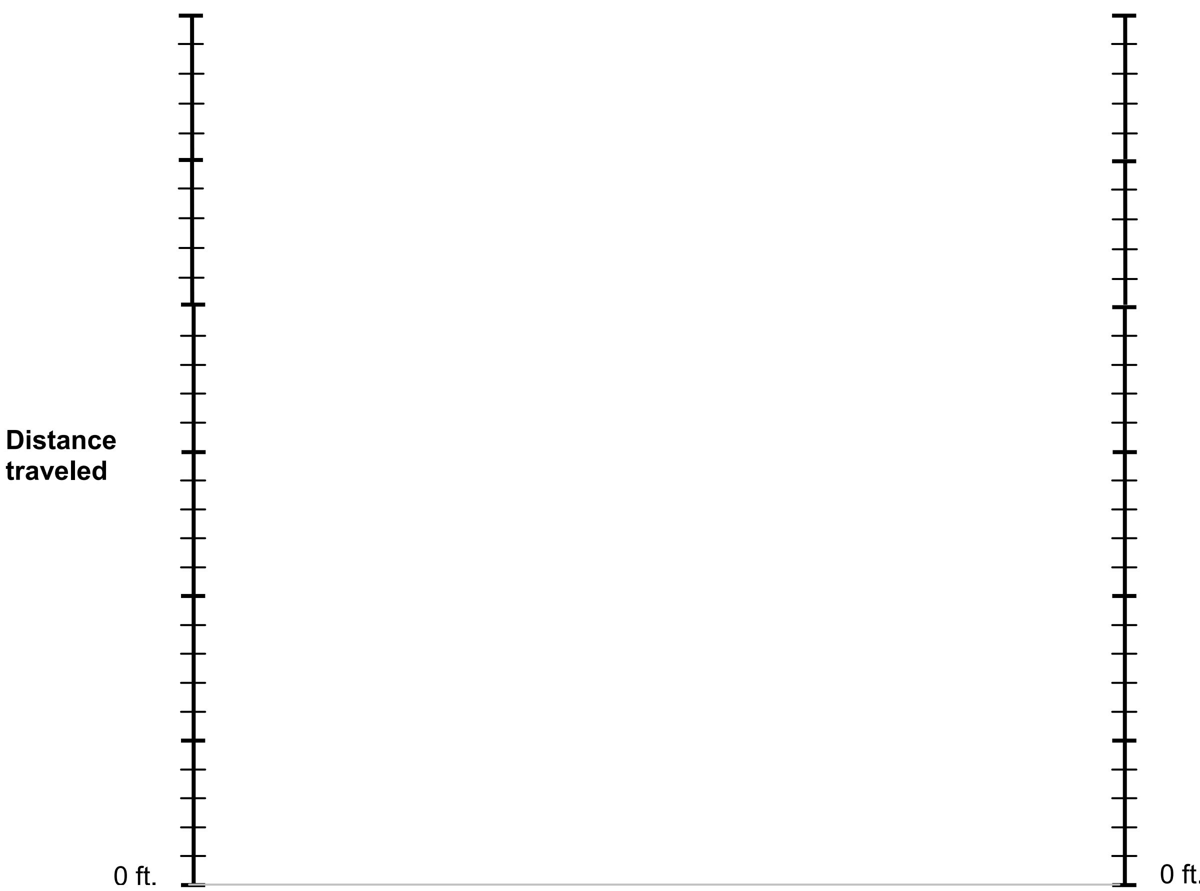
Name



Name

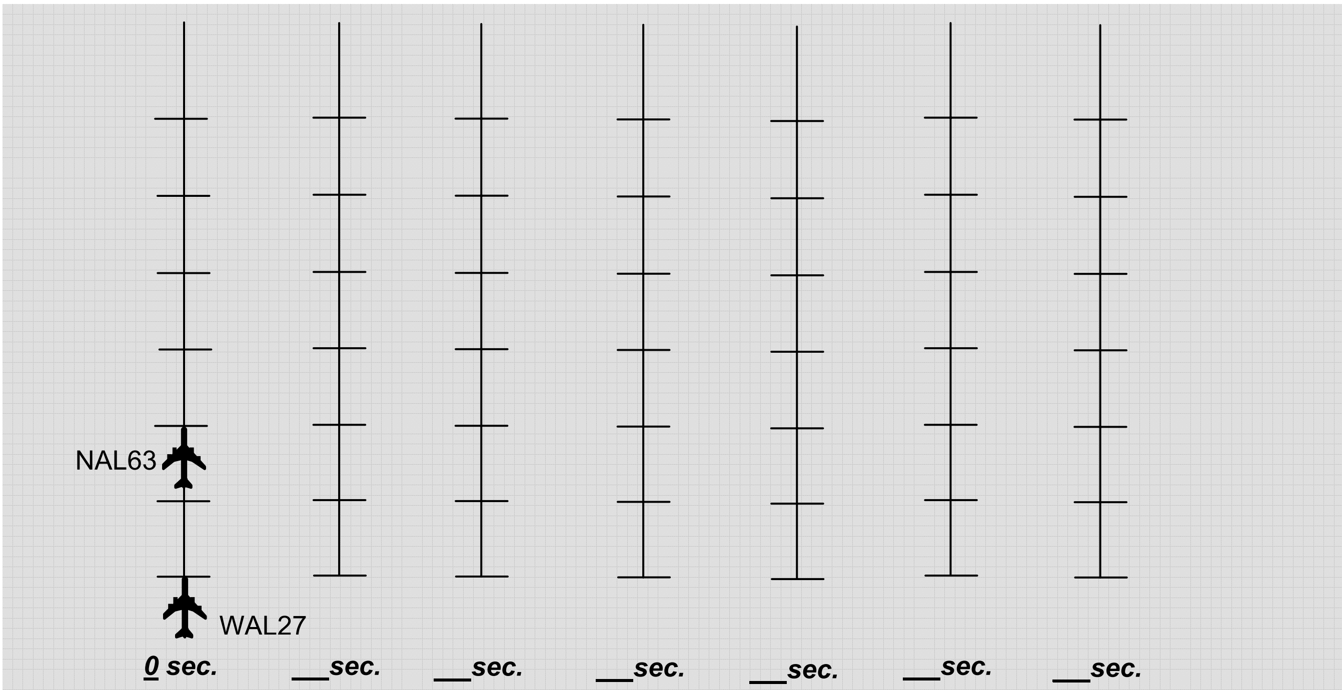
Lines and Grids





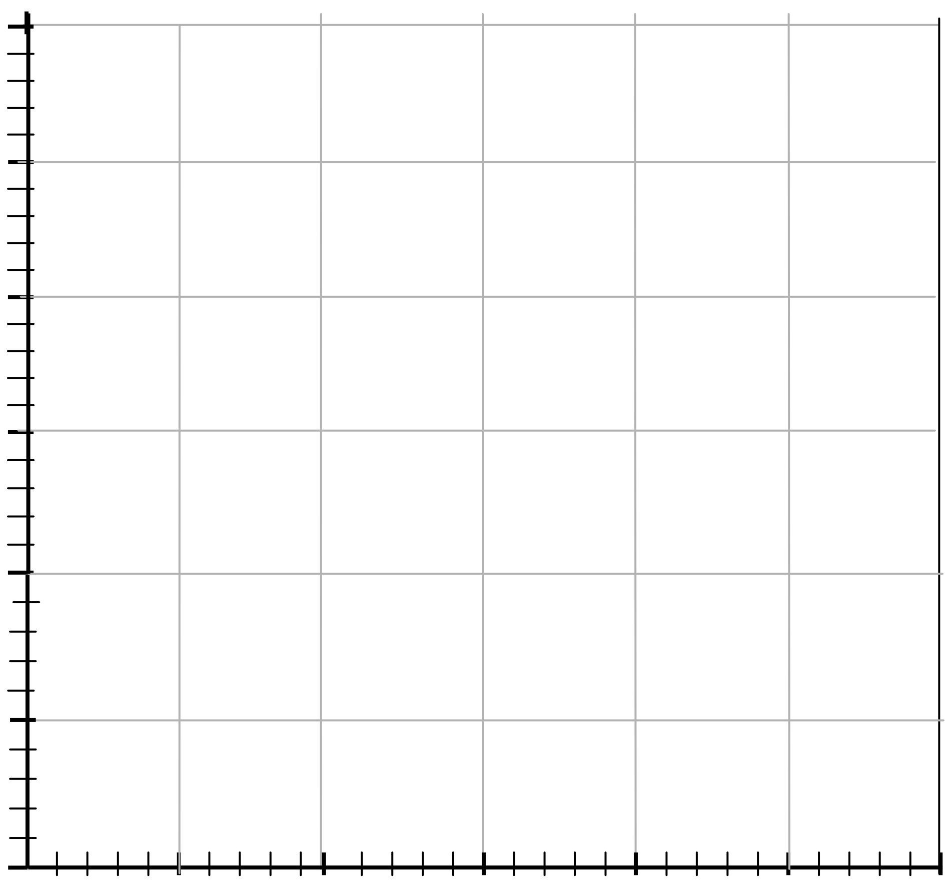
Name: ____________________

Lines and Grids



Distance traveled

0 ft.



0 secs.

Time traveled

Name

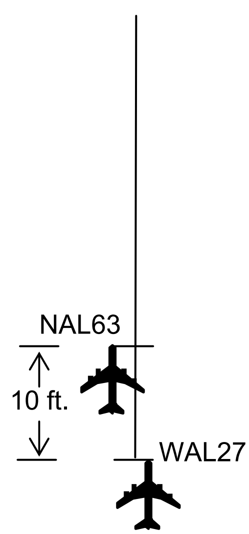
Set Up the Experiment

1. Use sidewalk chalk (or masking tape) to lay out 1 jet route as shown below. The route should be at least 20 feet long. It is better to make the route 25-30 feet long.

2. The speed of Flight WAL27 is 1/2 foot/second. Stand at the beginning of the jet route. Place a mark (or a piece of masking tape) every 1/2 foot (every 6 inches) along the **right** side of the jet route. This will guide the pilot as he or she steps down the jet route.

3. The speed of Flight NAL63 is 1/4 foot/second. Stand at the beginning of the jet route. Place a mark (or a piece of masking tape) every 1/4 foot (every 3 inches) along the **left** side of the jet route. This will guide the pilot as he or she steps down the jet route.

4. On the jet route, place and label a longer chalk mark (or longer piece of masking tape) at the following positions:
   5 feet from the start, **10 feet** from the start, **15 feet** from the start, **20 feet** from the start, and **25 feet** from the start.



Name

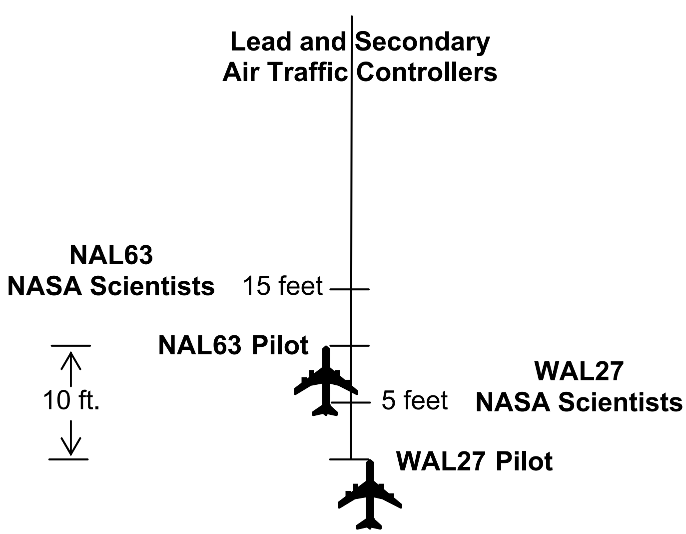
Conduct the Experiment

1. **Review your prediction.**

   How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with Flight NAL63?

2. **Take your position.** Circle your role in the diagram and in the following list:

   Lead Air Traffic Controller: Give the command "Take your ready positions."
   Pilots: Position yourself at the start of the jet route on your side of the route.
   Secondary Controller: Take your data sheet, measuring tape, and pencil and go to your controller location as shown below.
   NASA Scientists:
   WAL27 Scientist—Take your data sheet and pencil and go to your first observation position at the 5-foot line on your side of the route as shown below.
   NAL63 Scientist—Take your data sheet and pencil and go to your first observation position at the 15-foot line on your side of the route as shown below.



____
Name

3. **Get ready to begin.** Circle your role in the following list:

Lead Air Traffic Controller: Give the command “Set.”

Pilots: Prepare to step down the jet route. You may want to practice first. It helps to keep one foot on each side of the jet route.

NASA Scientists: Get ready to measure and record the information on the data sheet.

4. **Begin the experiment.** Circle your role in the following list:

Lead Air Traffic Controller: Give the command “Ready.” Start your stopwatch and count the seconds aloud, “One, two, three…” and so on.

Pilots: Take your first step on count “One.” Each second, take one step to the next timing mark.

NASA Scientists:

WAL27 Scientist—Record the time your aircraft arrives at the 5-foot line. Stay ahead of the pilot and record the time your aircraft arrives at the 10-foot line, the 15-foot line, and the point where the Controller says, “Halt.”

NAL63 Scientist—Record the time your aircraft arrives at the 15-foot line. Stay ahead of the pilot and record the time your aircraft arrives at the point where the Controller says, “Halt.”

5. **End the experiment.** Circle your role in the following list:

Secondary Controller: When Flight WAL27 catches up to Flight NAL63, give the command “Halt.”

Lead Air Traffic Controller: Stop counting the seconds when you hear “Halt.”

Pilots: Stop and remain where you are on the jet route when you hear “Halt.”

NASA Scientists: Record the “Halt” time.

Name

Data Sheet

a. Fill in this table:

Flight Number	Speed	Distance from the Start of the Jet Route
WAL27		
NAL63		

b. On the picture below, circle your job title. Notice the data you need to record.

c. During Experiments 1, 2, and 3, record your data.

1	2	3

Air Traffic Controllers

30’

25’

20’

15’

10’

5’

0’

"HALT" Time (NAL63)

1	2	3

Time (seconds)

1	2	3

NASA Scientist: NAL63
Start recording here.

START
Pilot: NAL63

"HALT" Time (WAL27)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

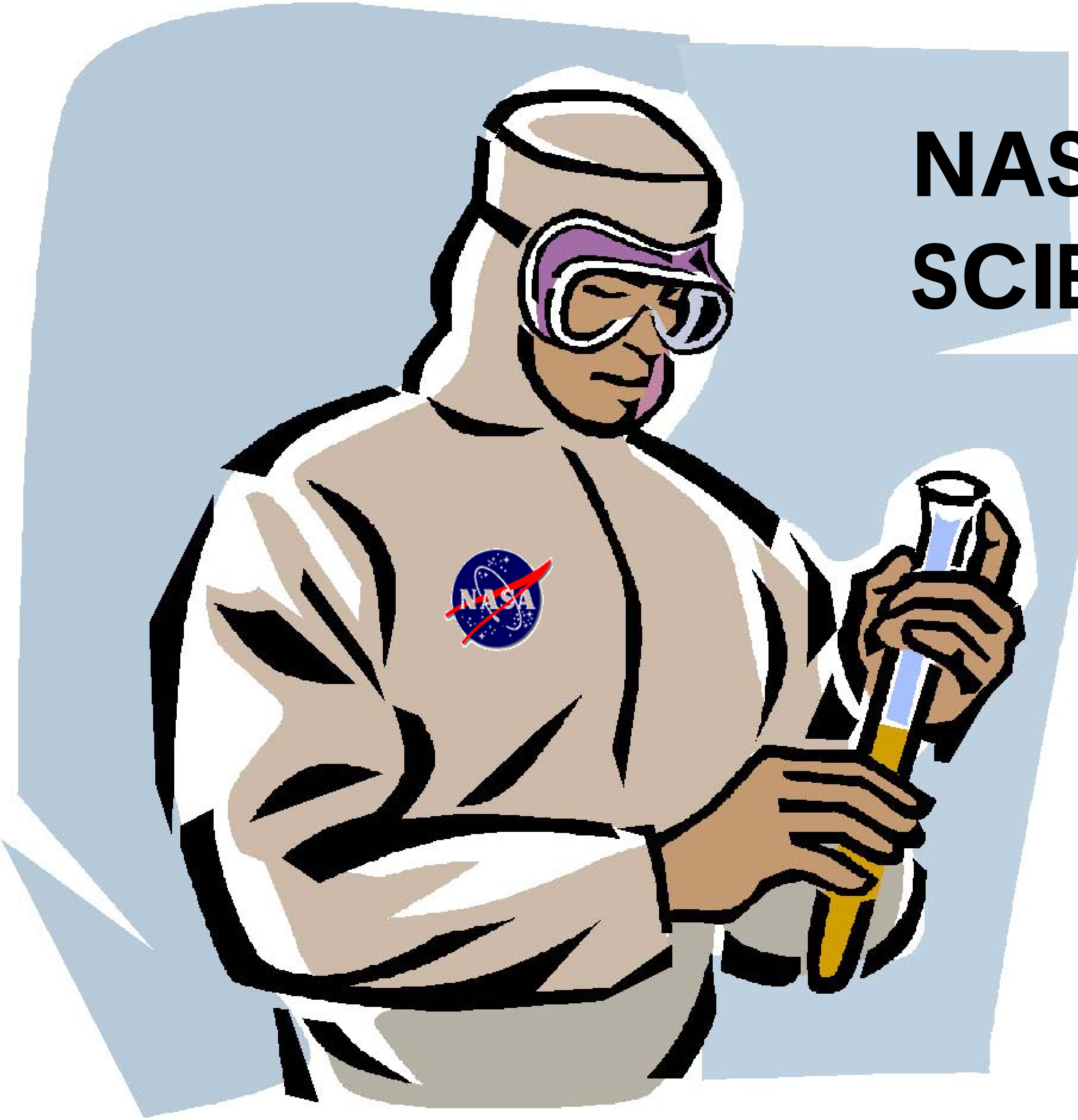
NASA Scientist: WAL27
Start recording here.

START
Pilot: WAL27

PILOT

AIR TRAFFIC CONTROLLER

NASA SCIENTIST

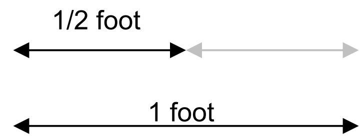


Name

How Much Time for the Trailing Plane to Close the Gap?
(Count Feet and Seconds to Find the Answer)

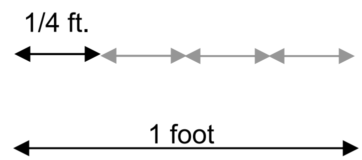
The speed of Flight WAL27 is 1/2 foot per second.
That means the plane travels 1/2 foot in 1 second.

So Flight WAL27 travels **1 foot in 2 seconds**.



The speed of Flight NAL63 is 1/4 foot per second.
That means the plane travels 1/4 foot in 1 second.

So Flight NAL63 travels **1 foot in 4 seconds**.



At the start of the problem, Flight WAL27 is 10 feet behind Flight NAL63.

1. Fill in the given table to see how many seconds it will take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63. That is, after how many seconds will each plane be at the same place at the same time? ________________

After you fill in the table, answer the following questions:

____
Name

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? __________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? _______

____

4. Compare your answer to Question 3 with your answer to Question 2.

____

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

____

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

____

Name

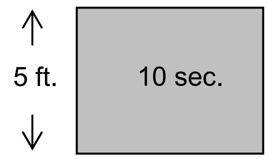
Flight WAL27		Flight NAL63	
How many feet from the start?	How many seconds?	How many feet from the start?	How many seconds?
0	0		
1	2		
2			
3			
4			
5			
6			
7			
8			
9			
10		10	0
11		11	4
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

Name

How Much Time for the Trailing Plane to Close the Gap?
(Draw Blocks to Find the Answer)

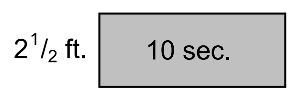
The speed of Flight WAL27 is 1/2 foot per second.
In 1 second, the plane travels 1/2 foot.
In 10 seconds, the plane travels 10 x 1/2 foot.
That is, Flight WAL27 travels 5 feet in 10 seconds.

The height of this block represents 5 feet, the distance Flight WAL27 travels in 10 seconds.



The speed of Flight NAL63 is 1/4 foot per second.
In 1 second, the plane travels 1/4 foot.
In 10 seconds, the plane travels 10 x 1/4 foot.
That is, Flight NAL63 travels $2^{1}/_{2}$ feet in 10 seconds.

The height of this block represents $2^{1}/_{2}$ feet, the distance Flight NAL63 travels in 10 seconds.



At the start of the problem, Flight WAL27 is 10 feet behind Flight NAL63.

Now you will use blocks and dots to plot the position of each plane as it travels along the jet route.

Look at the top picture on page 5. The two planes are shown on the jet route at 0 seconds, the time when the problem begins.

We also give you five copies of the jet route to make it easier for you to plot the positions of the planes at 10 seconds, 20 seconds, 30 seconds, and so on.

Name

Now look at the “10 seconds” jet route.

At the start of the problem, Flight NAL63 had already traveled 10 feet.

After 10 seconds, Flight NAL63 has traveled $2^{1}/_{2}$ more feet.

So the plane has traveled $12^{1}/_{2}$ feet.

An **O** shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 starts at 0 feet.

After 10 seconds, Flight WAL27 has traveled 5 feet.

An **X** shows the position of Flight WAL27 after 10 seconds.

Next look at the bottom picture on page 5. It shows a bar graph made of blocks.

A block shows the position of Flight NAL63 after 10 seconds.

A block shows the position of Flight WAL27 after 10 seconds.

The blocks are connected with a line marked “10 seconds.”

Now it’s your turn to draw and connect.

- ◻ On the “20 seconds” jet route, draw an **O** to show the position of Flight NAL63 after 20 seconds.
- ◻ On the bottom picture, draw a block to show the position of Flight NAL63 after 20 seconds.
- ◻ On the “20 seconds” jet route, draw an **X** to show the position of Flight WAL27 after 20 seconds.
- ◻ On the bottom picture, draw a block to show the position of Flight WAL27 after 20 seconds.

Name: ______________________

- ☐ Connect your blocks with a line marked "20 seconds."

- ☐ Draw an **O** and an **X** on the "30 seconds" jet route.
- ☐ Draw and connect blocks at 30 seconds.

- ☐ Keep going until Flight WAL27 catches up to Flight NAL63. That is, keep going until each plane is in the same place at the same time.

When you are done, answer these questions.

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? ____________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ____________

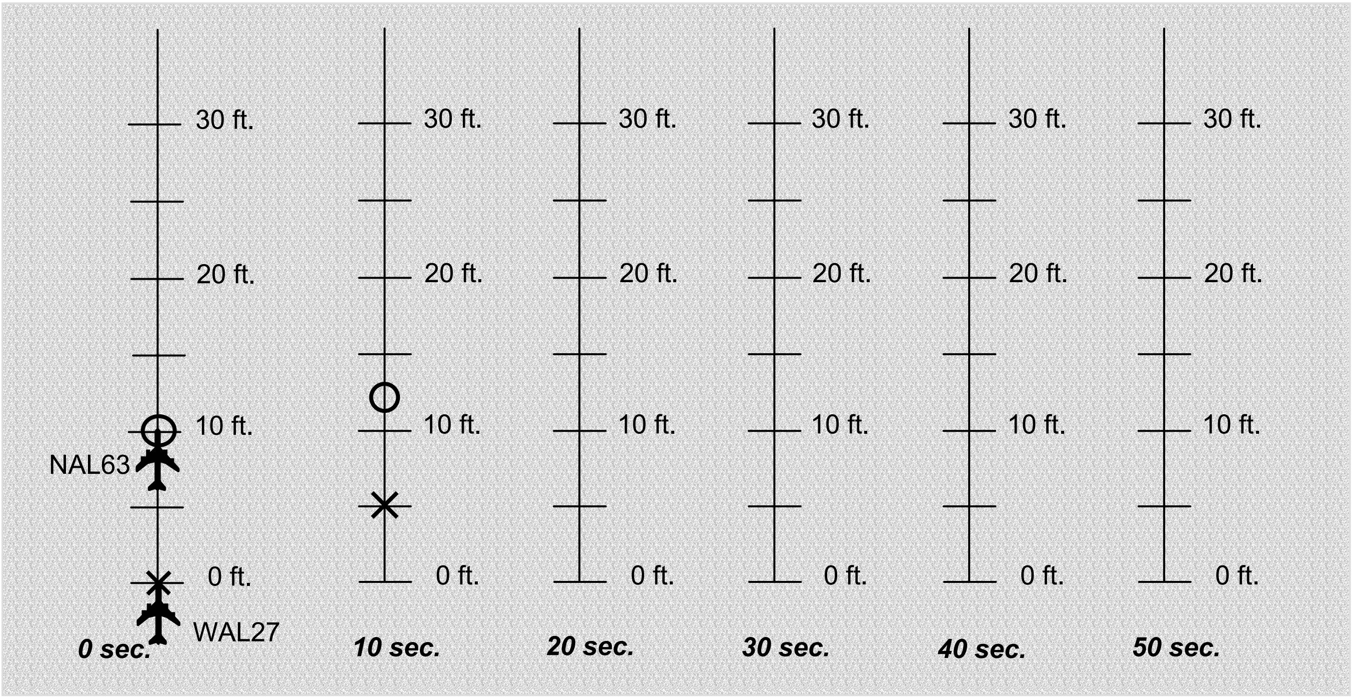
3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? ________
____

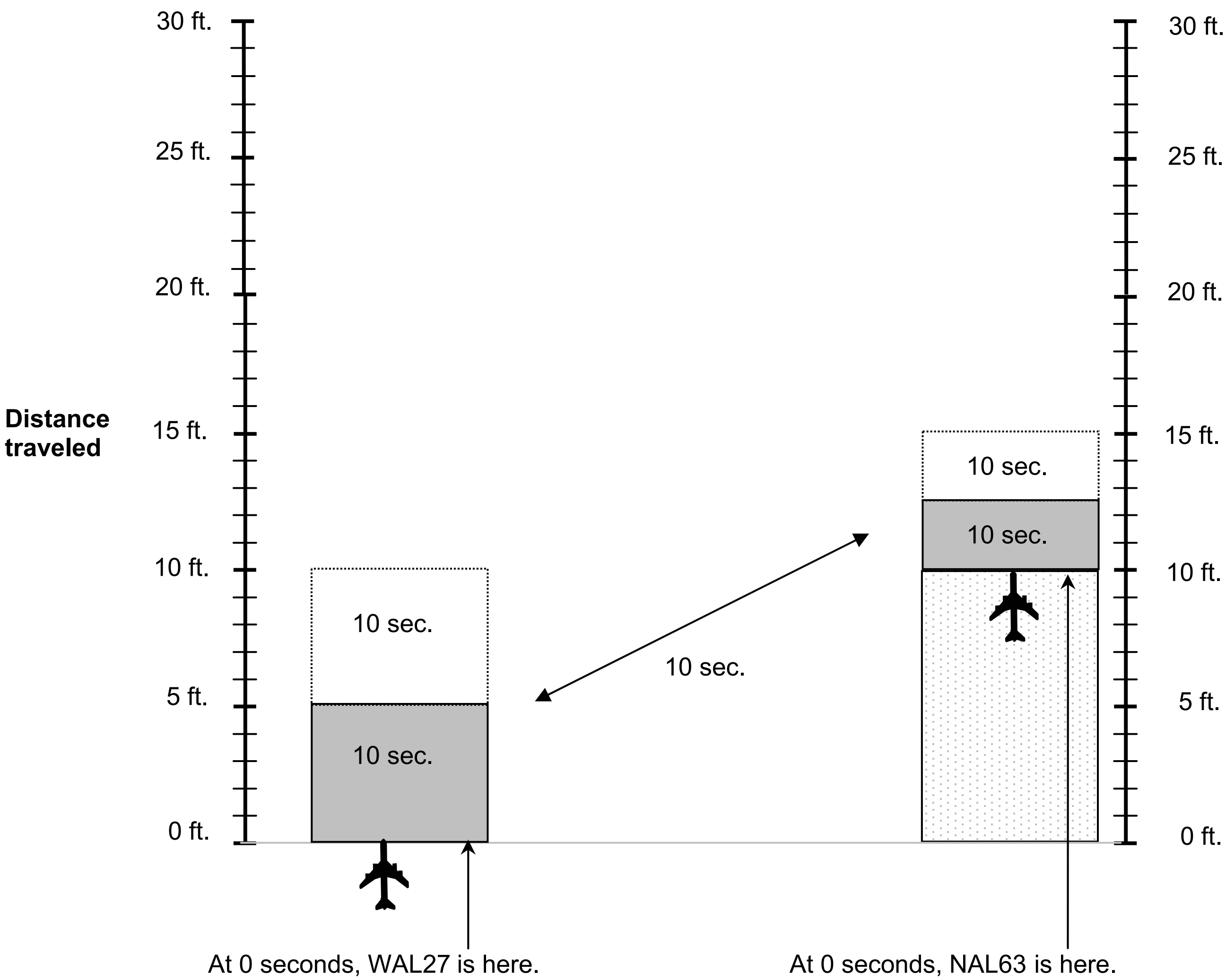
4. Compare your answer to Question 3 with your answer to Question 2.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

Name


30 ft.
20 ft.
10 ft.
0 ft.
NAL63
WAL27
0 sec.
10 sec.
20 sec.
30 sec.
40 sec.
50 sec.


Distance traveled
30 ft.
25 ft.
20 ft.
15 ft.
10 ft.
5 ft.
0 ft.
10 sec.
10 sec.
10 sec.
10 sec.
10 sec.
At 0 seconds, WAL27 is here.
At 0 seconds, NAL63 is here.

____

Name

How Much Time for the Trailing Plane to Close the Gap?
(Plot Points on Lines to Find the Answer)

The speed of Flight WAL27 is 1/2 foot per second.
In 1 second, the plane travels 1/2 foot.
In 10 seconds, the plane travels 10 x 1/2 foot.
That is, Flight WAL27 travels 5 feet in 10 seconds.

The speed of Flight NAL63 is 1/4 foot per second.
In 1 second, the plane travels 1/4 foot.
In 10 seconds, the plane travels 10 x 1/4 foot.
That is, Flight WAL27 travels $2\,^{1}/_{2}$ feet in 10 seconds.

You will use an **X** to show the position of Flight WAL27.
You will use an **O** to show the position of Flight NAL63.

Look at the top picture on page 4. The two planes are shown on the jet route at 0 seconds, the time when the problem begins.
We also give you five copies of the jet route to make it easier for you to plot the positions of the planes at 10 seconds, 20 seconds, 30 seconds, and so on.

Now look at the "10 seconds" jet route.

At the start of the problem, Flight NAL63 had already traveled 10 feet.
After 10 seconds, Flight NAL63 has traveled $2^{1}/_{2}$ more feet.
So the plane has traveled $12^{1}/_{2}$ feet.
An **O** shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 starts at 0 feet.
After 10 seconds, Flight WAL27 has traveled 5 feet.
An **X** shows the position of Flight WAL27 after 10 seconds.

____
Name

Next look at the bottom picture on page 4. It shows two vertical line graphs.

An **O** shows the position of Flight NAL63 after 10 seconds.
An **X** shows the position of Flight WAL27 after 10 seconds.
The **X** and **O** are connected with a line marked “10 seconds.”

Now it’s your turn to draw and connect.

- □ On the “20 seconds” jet route, draw an **O** to show the position of Flight NAL63 after 20 seconds.
- □ On the bottom picture, draw an **O** to show the position of Flight NAL63 after 20 seconds.

- □ On the “20 seconds” jet route, draw an **X** to show the position of Flight WAL27 after 20 seconds.
- □ On the bottom picture, draw an **X** to show the position of Flight WAL27 after 20 seconds.

- □ On the bottom picture, connect the **O** and the **X** with a line marked “20 seconds.”

- □ Draw an **O** and an **X** on the “30 seconds” jet route.
- □ On the bottom picture, draw and connect an **O** and an **X** to show the position of each plane after 30 seconds.

- □ Keep going until Flight WAL27 catches up to Flight NAL63. That is, keep going until each plane is in the same place at the same time.

Name

When you are done, answer these questions.

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? ________________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ________________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? ________

____

4. Compare your answer to Question 3 with your answer to Question 2.

____

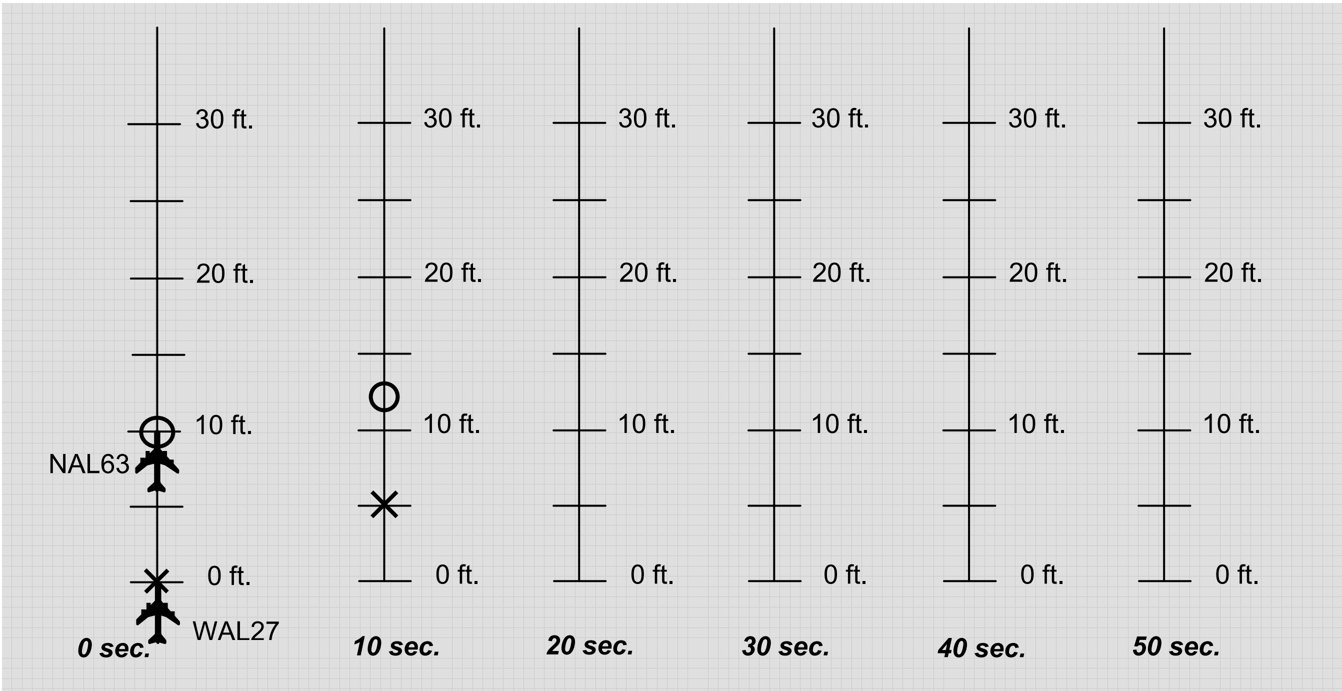
5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

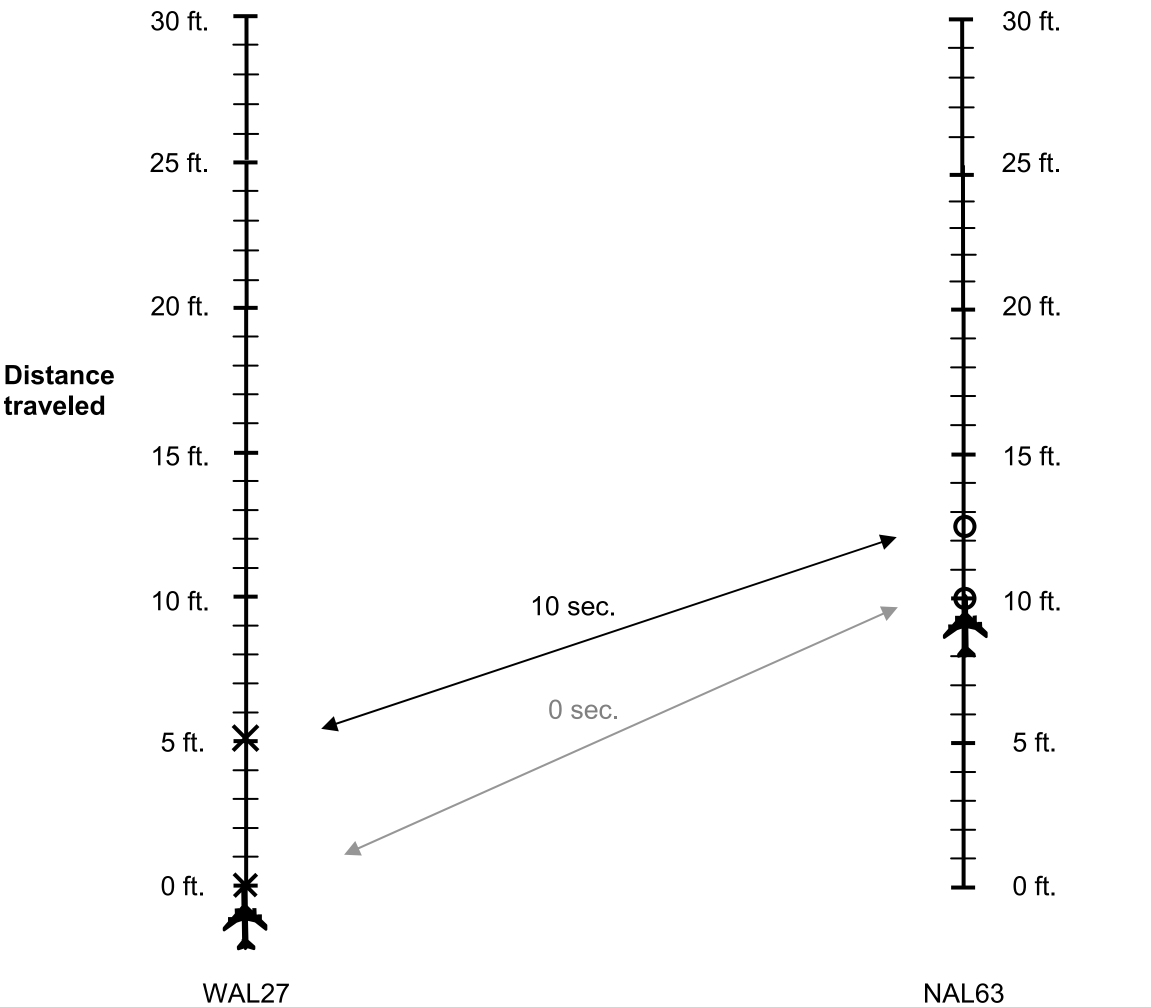
____

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

____

Name


30 ft.
20 ft.
10 ft.
0 ft.
NAL63
WAL27
0 sec.
10 sec.
20 sec.
30 sec.
40 sec.
50 sec.


Distance traveled
30 ft.
25 ft.
20 ft.
15 ft.
10 ft.
5 ft.
0 ft.
10 sec.
0 sec.
WAL27
NAL63

____
Name

How Much Time for the Trailing Plane to Close the Gap?
(Plot Points on a Grid to Find the Answer)

The speed of Flight NAL63 is 1/4 foot per second.

In 1 second, the plane travels 1/4 foot.

In 10 seconds, Flight WAL27 plane travels $2\frac{1}{2}$ feet.

At the start of the problem, Flight NAL63 had already traveled 10 feet.

After 10 seconds, the plane has traveled $2\frac{1}{2}$ more feet.

So after **10** seconds, Flight NAL63 is at $\mathbf{12\frac{1}{2}}$ feet on the jet route.

On the “10 seconds” jet route on page 4, we represent that information with an **O** at $12\frac{1}{2}$ feet.

On the grid on page 4, we represent that information with the point (**10**, $\mathbf{12\frac{1}{2}}$).

The **O** at the point (10, $12\frac{1}{2}$) shows that after 10 seconds, Flight NAL63 is at $12\frac{1}{2}$ feet.

The speed of Flight WAL27 is 1/2 foot per second.

In 1 second, the plane travels 1/2 foot.

In 10 seconds, Flight WAL27 travels 5 feet.

Flight WAL27 starts at 0 feet.

After 10 seconds, the plane has traveled 5 feet.

So after **10** seconds, Flight WAL27 is at **5** feet on the jet route.

On the “10 seconds” jet route, we represent that information with an **X** at 5 feet.

On the grid, we represent that information with the point (**10**, **5**).

The **X** at the point (10, 5) shows that after 10 seconds, Flight WAL27 is at 5 feet.

Name

Now it's your turn to plot points on the routes and on the grid on page 4.

On the route and on the grid, put an **O** at the point that shows the position of Flight NAL63 after 20 seconds.
On the route and on the grid, put an **X** at the point that shows the position of Flight WAL27 after 20 seconds.

Keep going until Flight WAL27 catches up to Flight NAL63. That is, keep going until each plane is in the same place at the same time.

When you have finished plotting points, answer these questions.

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? ________________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ________________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? ________
____

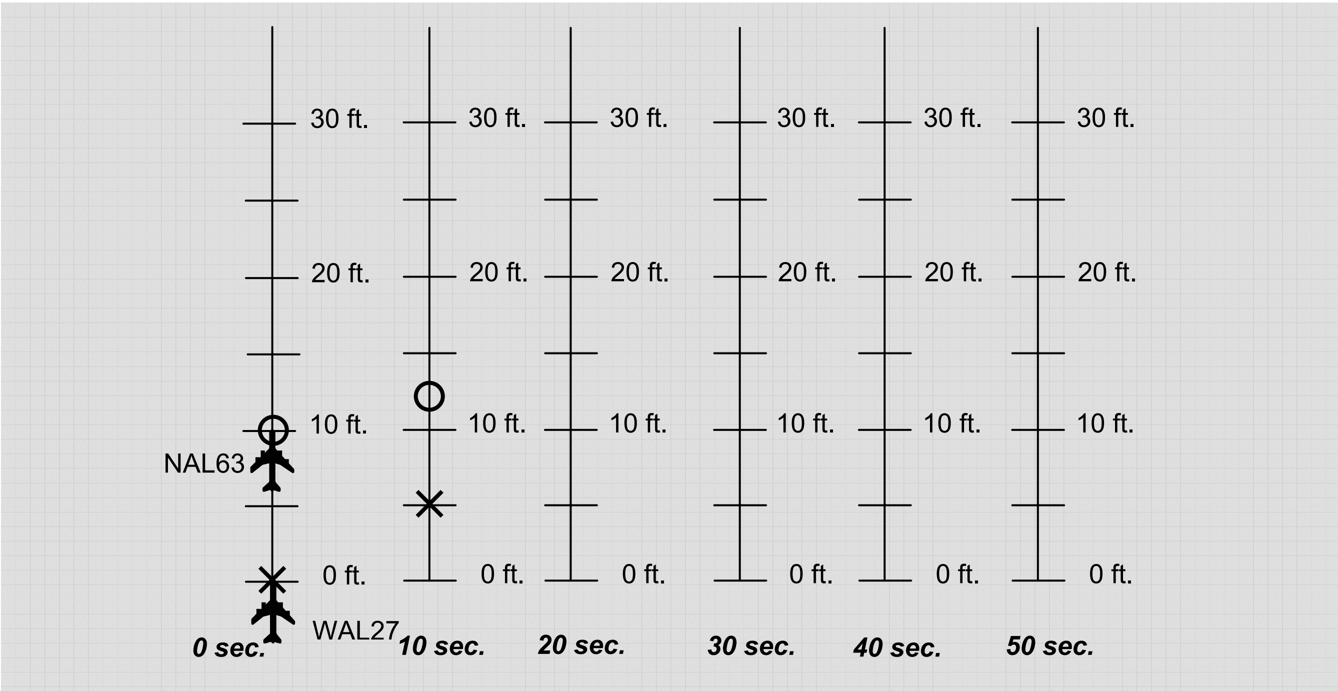
Name

4. Compare your answer to Question 3 with your answer to Question 2.

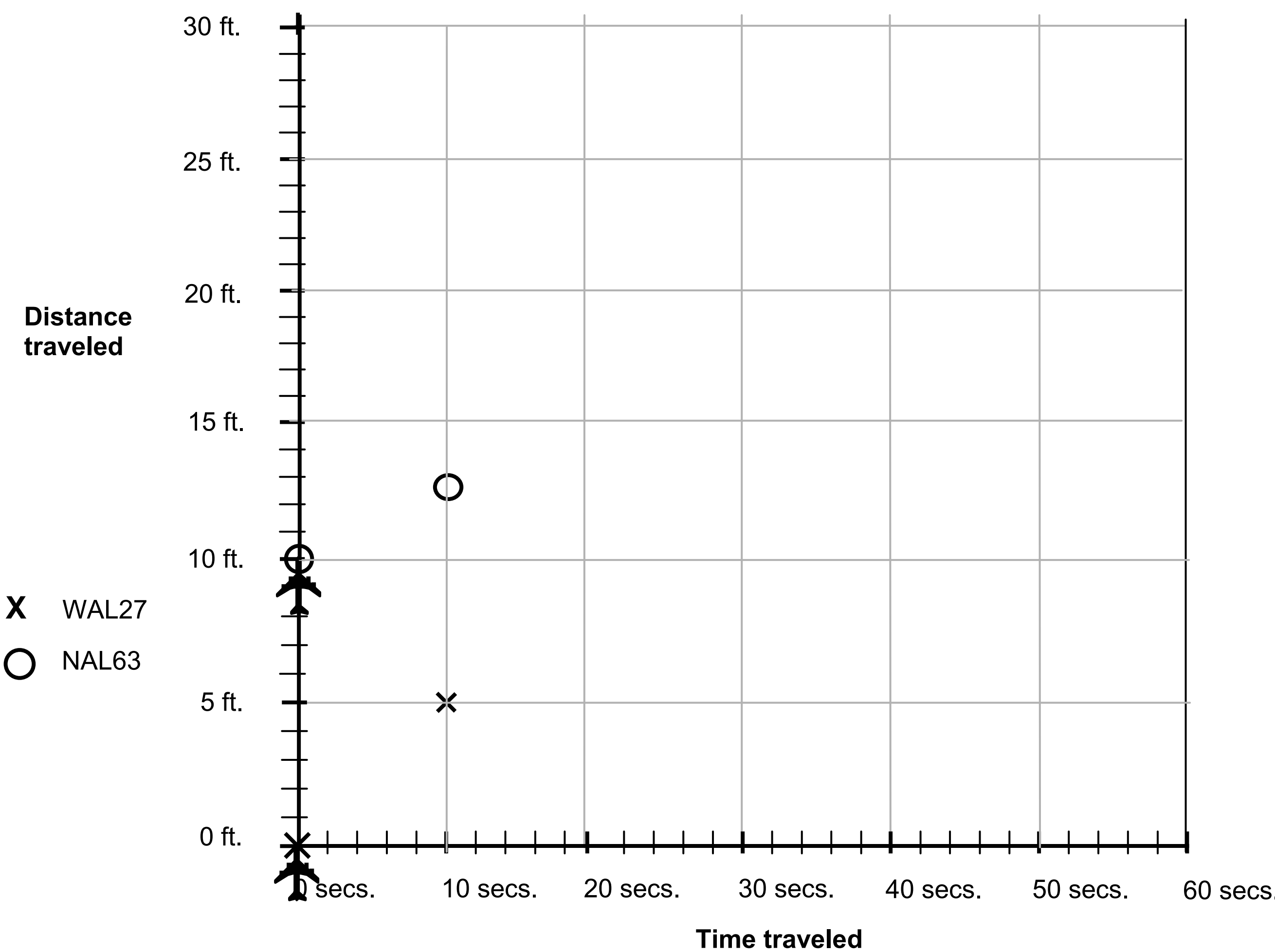
5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

6. Compare your answer to Problem 5 with your answer to Problem 3. With the new faster speed, how many seconds will it take to close the gap?

Name



30 ft.
20 ft.
10 ft.
0 ft.
NAL63
WAL27
0 sec.
10 sec.
20 sec.
30 sec.
40 sec.
50 sec.



Distance traveled
30 ft.
25 ft.
20 ft.
15 ft.
10 ft.
5 ft.
0 ft.
X WAL27
NAL63
0 secs.
10 secs.
20 secs.
30 secs.
40 secs.
50 secs.
60 secs.
Time traveled

____
Name

$d = r \bullet t$

Derive the Distance-Rate-Time Formula

The speed of Flight WAL27 is 0.5 feet per second.

In 1 second, the plane travels 0.5 feet.

In 2 seconds, the plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, the plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

1. In 4 seconds, the plane travels _____________ $\times$ _________ = ____ feet.
2. In 5 seconds, the plane travels _____________ $\times$ _________ = ____ feet.
3. In 6 seconds, the plane travels _____________ $\times$ _________ = ____ feet.
4. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds?

____

____

One way to find the distance is to multiply the plane's speed by 14 seconds, like this:

0.5 feet/second $\times$ 14 seconds = 7 feet

This suggests the following rule:

To find the distance traveled, multiply the speed by the time traveled.

In mathematics and science, we often say "rate" instead of "speed."

Then we can write the rule like this:

distance = **r**ate $\times$ **t**ime

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$\mathbf{d = r \bullet t}$$

5. Use the formula to find the distance traveled by Flight WAL27 in 30 seconds.
   In 30 seconds, Flight WAL27 travels ___________ feet.

Name

d = r • t

6. The speed of Flight NAL63 is 1/4 foot per second.

Use the formula **d = r • t** to find the distance traveled by Flight NAL63 in 30 seconds.

In 30 seconds, Flight NAL63 travels __________ feet.

Name ______________________

t = d / r

Use the Distance-Rate-Time Formula

The speed of Flight WAL23 is 0.5 feet per second.

In 1 second, the plane travels 0.5 feet.

In 2 seconds, the plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, the plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

To find the distance traveled after t seconds, we multiply the rate by the time:

distance traveled = **r**ate of travel $\times$ **t**ime traveled

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$\mathbf{d = r \bullet t}$$

You can use this formula to help you find the number of seconds for Flight WAL27 to catch up to Flight NAL63.

To do this, you will calculate distances for different given times (10 seconds, 20 seconds, 30 seconds, and so on). In particular, you will:

- ◻ Find the distance traveled by each plane at a given time.
- ◻ Add that distance to the plane's starting position to find the plane's new position on the jet route.
- ◻ Compare the new position of each plane on the jet route.

You will need to know the speed of each plane:

The speed of Flight WAL23 is 0.5 feet per second.

The speed of Flight NAL63 is 0.25 feet per second.

We begin by using the formula to calculate distances at 10 seconds.

Name

t = d / r

At 10 seconds:

- Find the distance traveled by each plane at 10 seconds:

Flight NAL63: d = r · t

d = 0.25 feet/second · 10 seconds = 2.5 feet

Flight WAL27: d = r · t

d = 0.5 feet/second · 10 seconds = 5 feet

- Find the new position of each plane at 10 seconds:

Flight NAL63: new position = starting position + distance traveled

new position = 10 feet + 2.5 feet = 12.5 feet

Flight WAL27: new position = starting position + distance traveled

new position = 0 feet + 5 feet = 5 feet

- Compare the positions of each plane at 10 seconds.

Are the two planes in the same position on the jet route? No.

Flight NAL63 is at 12.5 feet and Flight WAL 23 is at 5 feet.

Now it's your turn to use the distance-rate-time formula.

____
Name

t = d / r

At 20 seconds:

- ◻ Find the distance traveled by each plane at 20 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

- ◻ Find the new position of each plane at 20 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

- ◻ At 20 seconds, are the two planes in the same position on the jet route? Why or why not? ________________________________

  ____

At 30 seconds:

- ◻ Find the distance traveled by each plane at 30 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

- ◻ Find the new position of each plane at 30 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

- ◻ At 30 seconds, are the two planes in the same position on the jet route? Why or why not? ________________________________

  ____

At 40 seconds:

- ◻ Find the distance traveled by each plane at 40 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

____
Name

t = d / r

- ◻ Find the new position of each plane at 40 seconds:

  Flight NAL63 ____________ Flight WAL27 __________

- ◻ At 40 seconds, are the two planes in the same position on the jet route? Why or why not? ______________________________

  ____

Now answer these questions.

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? _______________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ______________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? _______

   ____

4. Compare your answer to Question 3 with your answer to Question 2.

   ____

Name

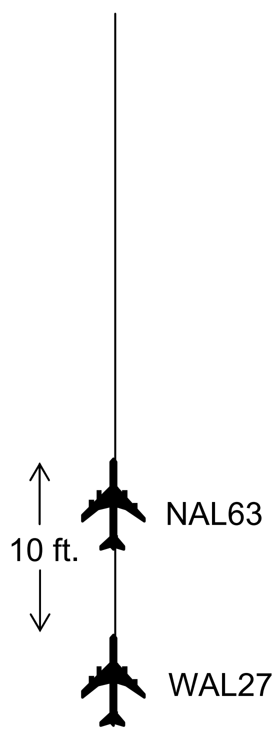
t = d / r

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

____

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

____



____
Name

How Much Time for the Trailing Plane to Close the Gap?
(Graph Two Linear Equations to Find the Answer)

We can use a linear equation to describe the position of an airplane that travels at a constant speed.

We begin with Flight WAL27.

The speed of Flight WAL27 is **1/2 foot per second**.
When the clock starts at 0 seconds, the plane is at **0 feet** on the jet route.
The position of Flight WAL27 is given by the equation $y = (1/2)x + 0$, which we can write like this:

$y = (1/2)x$ WAL27

Here:
- x = the time traveled (in seconds) and
- y = the number of feet traveled on the jet route

Now think about Flight NAL63.

The speed of Flight NAL63 is **1/4 foot per second**.
When the clock starts at 0 seconds, the plane is at **10 feet** on the jet route.
So the position of Flight NAL63 is given by this equation:

$y = (1/4)x + 10$ NAL63

Complete each table and use the ordered pairs to graph each line.

Name

WAL27 $y = (1/2)x$

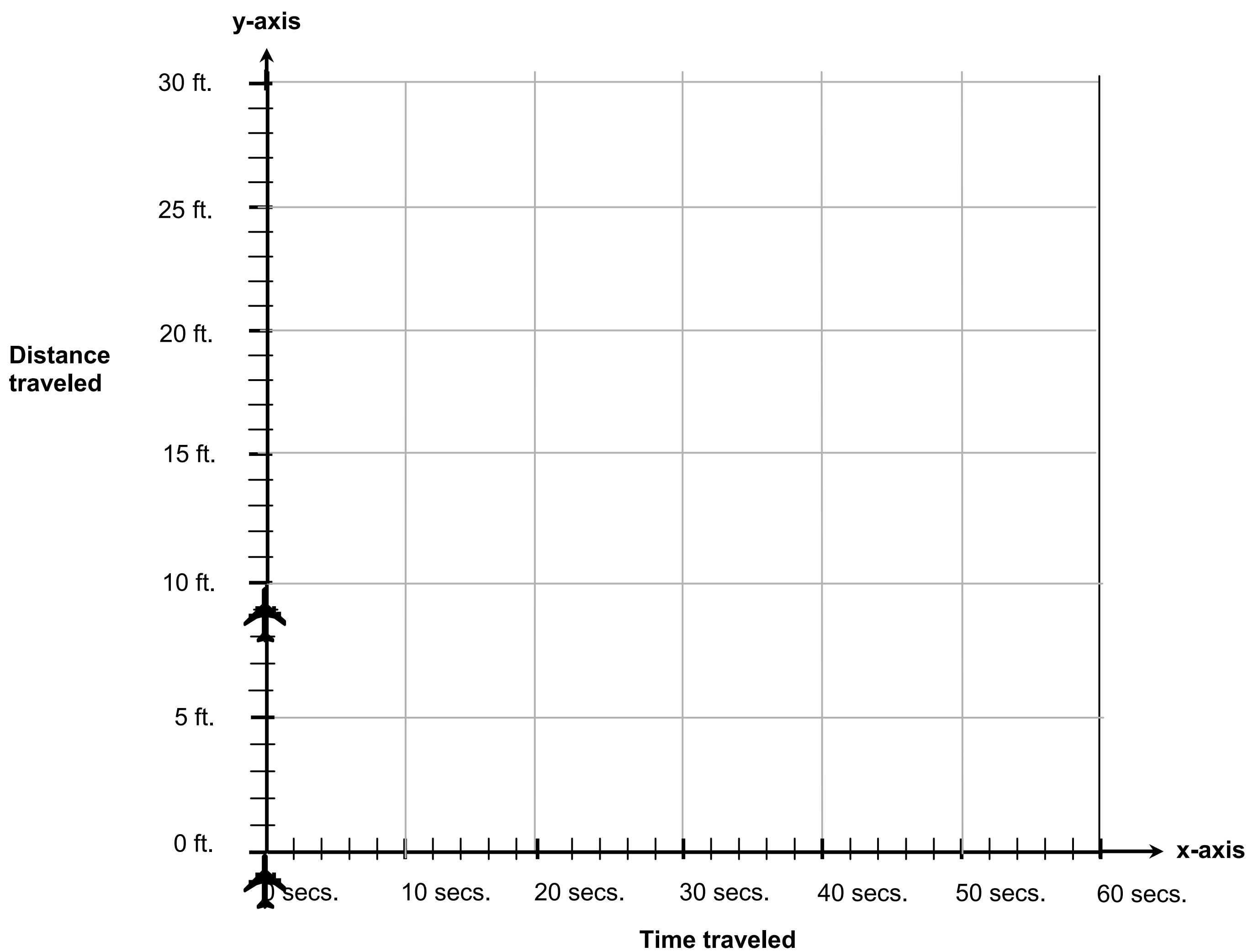
x	y
0	
10	
20	
30	
40	

NAL63 $y = (1/4)x + 10$

x	y
0	
10	
20	
30	
40	

Use a solid line for the graph of **Flight WAL27**.

Use a dotted line for the graph of **Flight NAL63**.



____
Name

Use your graphs to answer the following questions.

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? ____________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ____________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? ______

4. Compare your answer to Question 3 with your answer to Question 2.

____

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

____

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

____

7. Write the number that is the slope of the solid line representing Flight WAL27. ________

8. Write the number that is the slope of the dotted line representing Flight NAL63. ________

9. What information does the slope of each line tell you about each plane?

____

____
Name

After the Experiment

Now you will compare your prediction with the results of the experiment.

First, circle your role in the experiment:

Pilot of WAL27	NASA Scientist for WAL27	Lead Air Traffic Controller
Pilot of NAL63	NASA Scientist for NAL63	Other Air Traffic Controller

----

Take a look at your prediction.
How many seconds did you think it would take for Flight WAL27 to catch up to Flight NAL63? ____________________

Next look at the results of the experiment.
How many seconds did it take for Flight WAL27 to catch up to Flight NAL63? ____________________

Does your prediction match the experiment? Yes No

If your answer to the last question is No, why do you think your prediction and the experiment do not match? ____

____

Take another look at the problem.

The speed of Flight WAL27 was 1/2 foot per second.
The speed of Flight NAL63 was 1/4 foot per second.

What is the difference in speed between Flight WAL27 and Flight NAL63?
That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? ______________

____
Name

How fast is Flight WAL27 closing the gap between the planes?
That is, the distance between the planes changes how many feet each second?

What is the relationship between the difference in speeds and the speed at which Flight WAL27 is closing the gap?

The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes? With the new faster speed, how many seconds will it take to close the gap?

Now think about this general problem.

Two planes are traveling at different speeds on the same route.
The trailing plane is traveling faster than the leading plane.

Do you have enough information to predict how long it will take the trailing plane to catch up to the leading plane? Why or why not?

Name

Now suppose the difference in speeds is twice as great.

What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? Why?

___

___

Finally, suppose that the planes each travel at their original speeds, but the distance between the planes is twice as great.

What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? Why?

___

___

____
Name

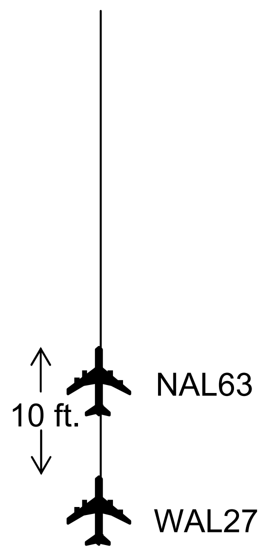
Posttest

In the picture below, two airplanes are flying on the same route.

The speed of Flight WAL27 is 1 foot/second.

The speed of Flight NAL63 is 1/4 foot/second.

At the start of the problem, Flight WAL27 is 10 feet behind Flight NAL63.



1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with Flight NAL63? __________________

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? __________________

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?____

____
Name

4. What is the relationship between the difference in speeds and the speed at which Flight WAL27 is closing the gap?

____

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes? With the new faster speed, how many seconds will it take to close the gap?

____

Now think about this general problem.

Two planes are traveling at different speeds on the same route.
The trailing plane is traveling faster than the leading plane.

6. Do you have enough information to predict how long it will take the trailing plane to catch up to the leading plane? Why or why not?

____

____

____
Name

7. Now suppose the difference in speeds is twice as great.
   What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? Why?

   ____

   ____

8. Finally, suppose that the planes each travel at their original speeds, but the distance between the planes is twice as great.
   What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? Why?

   ____

   ____

Airspace Systems – Predicting Air Traffic Conflicts

Curriculum Supplement 7

ANSWERS & EXPLANATIONS

A note on the organization of the Answers and Explanations:
In this Curriculum Supplement, most of the activities pose the same set of questions. The answers to those questions are introduced in the first part of this answer document. The remainder of the document is organized by activity and includes answers to individual activity questions, graphs, applications of the distance-rate-time formula, and discussions of the general problems posed in the analysis activity and the posttest.

The speed of Flight WAL27 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
The speed of Flight NAL63 is 1/4 foot/second, so the plane travels 1/4 foot in 1 second.
The planes are traveling on the same route.
At the start of the problem, the faster plane (Flight WAL27) is 10 feet behind the slower plane (Flight NAL63).

Since the trailing plane is traveling more rapidly than the leading plane, the trailing plane will catch up to the leading plane.

In particular:

- It will take 40 seconds for Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63.

- The speed of Flight WAL27 is 1/4 foot per second greater than the speed of Flight NAL63.

- The trailing plane closes the gap at a rate equal to the difference in the speeds of the two planes. So Flight WAL27 closes the 10-foot gap at a rate of 1/4 foot per second, the difference in the plane speeds.

- If the speed of WAL27 were increased to 3/4 foot/second, then the difference in the speeds of the two planes would be 1/2 foot/second. (That is double the difference of the original speeds.) Then WAL27 would close the 10-foot gap at a rate of 1/2 foot/second. (That is double the original closing rate.) It will take 20 seconds to close the gap.

Activity 7.3A—Count Feet and Seconds:

Flight WAL27 travels 1 foot in 2 seconds. Count by 2s to complete the table for Flight WAL27.

Flight NAL63 travels 1 foot in 4 seconds. Count by 4s to complete the table for Flight NAL63.

After 40 seconds, each plane will be 20 feet from the start and a conflict will occur.

1. Fill in the given table to see how many seconds it will take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63. That is, after how many seconds will each plane be at the same place at the same time? **40 seconds**

Flight WAL27		Flight NAL63	
How many feet from the start?	How many seconds?	How many feet from the start?	How many seconds?
0	0		
1	2		
2	**4**		
3	**6**		
4	**8**		
5	**10**		
6	**12**		
7	**14**		
8	**16**		
9	**18**		
10	**20**	10	0
11	**22**	11	4
12	**24**	**12**	**8**
13	**26**	**13**	**12**
14	**28**	**14**	**16**
15	**30**	**15**	**20**
16	**32**	**16**	**24**
17	**34**	**17**	**28**
18	**36**	**18**	**32**
19	**38**	**19**	**36**
20	**40**	**20**	**40**
21	**42**	**21**	**44**
22	**44**	**22**	**48**
23	**46**	**23**	**52**
24	**48**	**24**	**56**
25	**50**	**25**	**60**

Activity 7.3A (cont.)

2. What is the difference in speed between Flight WAL27 and Flight NAL63?
That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?
Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.
The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?
The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

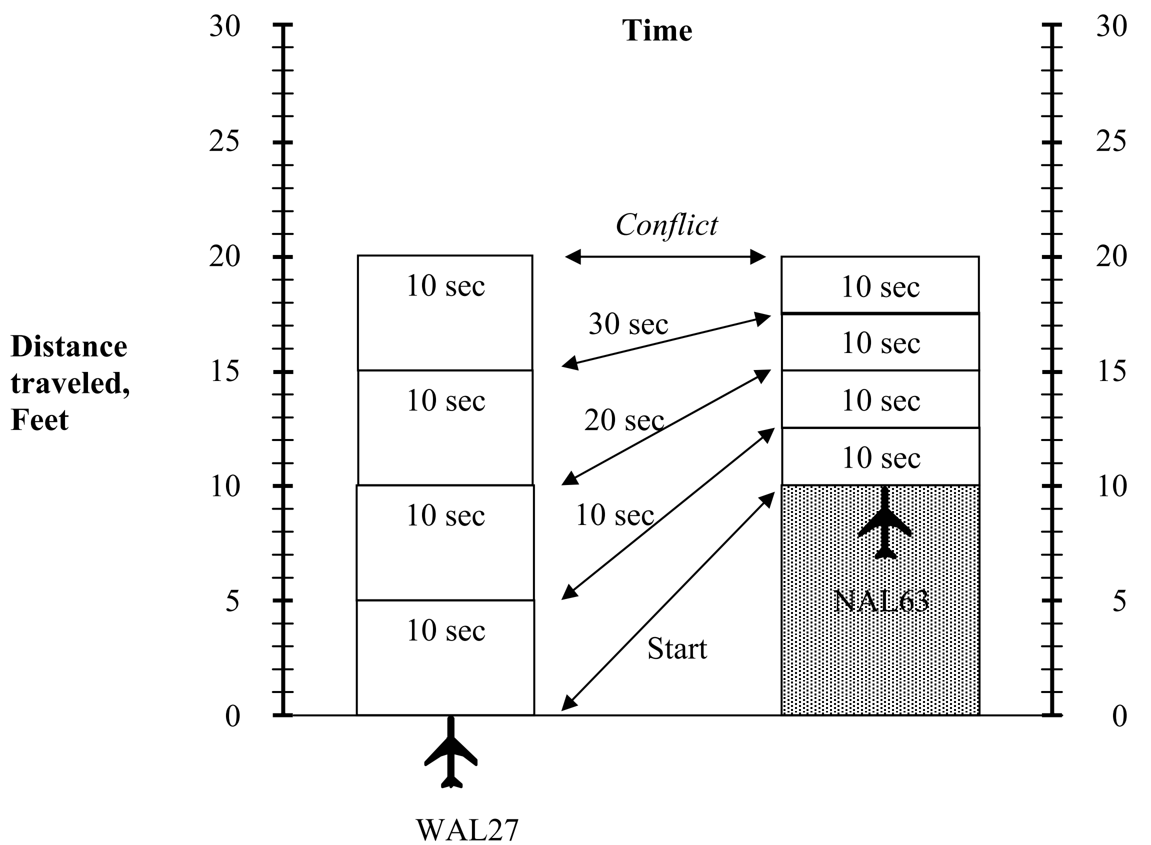
6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?
The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

Activity 7.3B—Stacking Blocks:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2\,^1/_2$ feet.

The following diagram shows a stack of 10-second blocks for each plane.
In the stack corresponding to Flight WAL27, each block represents 5 feet.
In the stack corresponding to Flight NAL63, each block represents $2\,^1/_2$ feet.

By adding a 5-foot block to Flight WAL27 and a $2\,^1/_2$–foot block to Flight NAL63 at the same time, students will see that each stack requires 4 blocks to reach the same height. When the stacks are the same height, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? **40 seconds**

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

Activity 7.3B (cont.)

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?

   Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.

   The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

   The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

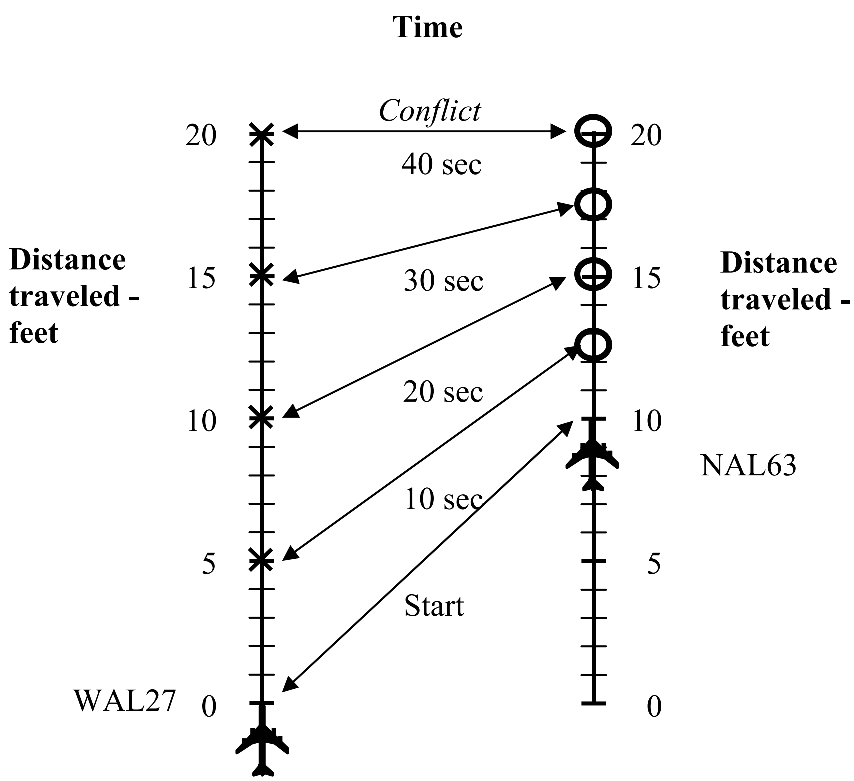
   The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

Activity 7.3C—Plot Points on Lines:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2\,^{1}/_{2}$ feet.

The following diagram shows the position of each plane at 10-second intervals.

By plotting a point for WAL27 and immediately plotting the corresponding point for Flight NAL63, students will see that when the points are the same distance from the start of the jet route, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? **40 seconds**

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

Activity 7.3C (cont.)

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?

   Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.

   The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?

   The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

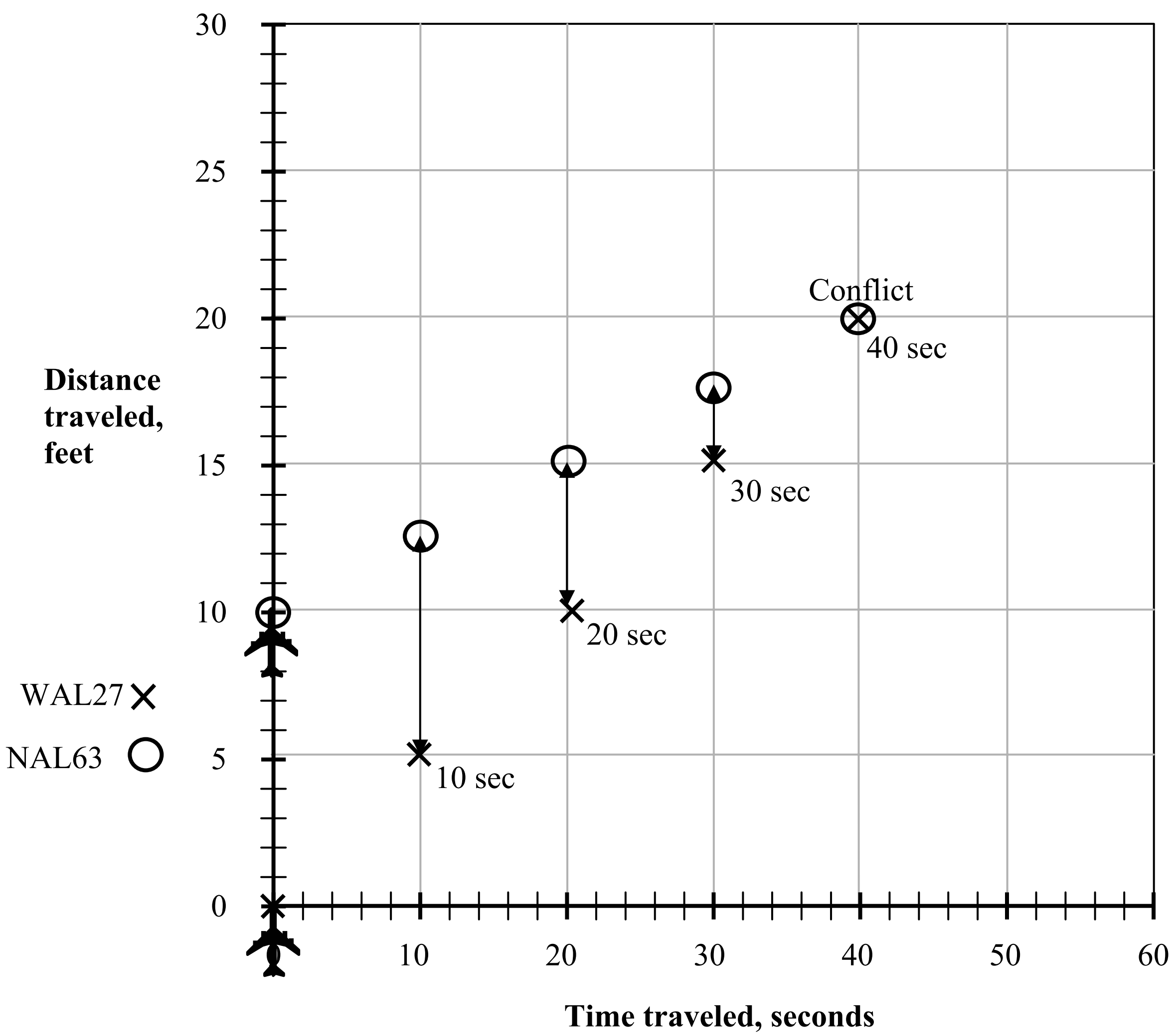
6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

   The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

Activity 7.3D—Plot Points on a Grid:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2\,^1/_2$ feet.

The following graph shows the position of each plane at 10-second intervals. By plotting a point for WAL27 and immediately plotting the corresponding point for Flight NAL63, students will see that when the points are the same distance from the start of the jet route, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? __**40 seconds**__

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?
   Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.
   The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?
   The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?
   The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

Activity 7.3E—Derive the Distance-Rate-Time Formula (Grades 6-8):

In 4 seconds, Flight WAL27 travels 0.5 feet/second x 4 seconds = 2.0 feet.

In 5 seconds, Flight WAL27 travels 0.5 feet/second x 5 seconds = 2.5 feet.

In 6 seconds, Flight WAL27 travels 0.5 feet/second x 5 seconds = 3.0 feet.

To find the distance Flight WAL27 travels in 14 seconds, multiply 0.5 feet/second by 14.

To find the distance Flight WAL27 travels in 30 seconds, multiply 0.5 feet/second by 30. The result is 15.0 feet.

To find the distance Flight NAL63 travels in 30 seconds, multiply 0.25 feet/second by 30. The result is 7.5 feet.

1. In 4 seconds, the plane travels **0.5 feet/second** × **4 seconds** = **2.0** feet.
2. In 5 seconds, the plane travels **0.5 feet/second** × **5 seconds** = **2.5** feet.
3. In 6 seconds, the plane travels **0.5 feet/second** × **6 seconds** = **3.0** feet.

4. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**

5. Use the formula to find the distance traveled by Flight WAL27 in 30 seconds.
   In 30 seconds, Flight WAL27 travels **15** feet.

6. The speed of Flight NAL63 is 1/4 foot per second.
   Use the formula **d = r • t** to find the distance traveled by Flight NAL63 in 30 seconds.
   In 30 seconds, Flight NAL63 travels **7.5** feet.

Activity 7.3F—Use the Distance-Rate-Time Formula:

Find the distance traveled by each plane in 20 seconds:

Flight WAL27: $d = r \cdot t = 0.5$ feet/second x 20 seconds = 10 feet

Flight NAL63: $d = r \cdot t = 0.25$ feet/second x 20 seconds = 5 feet

Find the new position of each plane after 20 seconds:

new position = starting position + distance traveled

Flight WAL27: new position = 0 feet + 10 feet = 10 feet

Flight NAL63: new position = 10 feet (10 ft headstart) + 5 feet = 15 feet

Compare the positions of the planes after 20 seconds:

The planes are *not* in the same position on the jet route.

Find the distance traveled by each plane in 30 seconds:

Flight WAL27: $d = r \cdot t = 0.5$ feet/second x 30 seconds = 15 feet

Flight NAL63: $d = r \cdot t = 0.25$ feet/second x 30 seconds = 7.5 feet

Find the new position of each plane after 30 seconds:

new position = starting position + distance traveled

Flight WAL27: new position = 0 feet + 15 feet = 15 feet

Flight NAL63: new position = 10 feet (10 ft headstart) + 7.5 feet = 17.5 feet

Compare the positions of the planes after 30 seconds:

The planes are *not* in the same position on the jet route.

Activity 7.3F (cont.)
Find the distance traveled by each plane in 40 seconds:

Flight WAL27: $d = r \cdot t$ = 0.5 feet/second x 40 seconds = 20 feet

Flight NAL63: $d = r \cdot t$ = 0.25 feet/second x 40 seconds = 10 feet

Find the new position of each plane after 40 seconds:

new position = starting position + distance traveled

Flight WAL27:	new position =	0 feet	+	20 feet	=	20 feet
Flight NAL63:	new position =	10 feet (10 ft headstart)	+	10 feet	=	20 feet

Compare the positions of the planes after 40 seconds:

The planes are in the same position on the jet route. The planes are in conflict.

At 20 seconds:

- □ Find the distance traveled by each plane at 20 seconds:
  Flight NAL63 **5 feet** Flight WAL27 **10 feet**

- □ Find the new position of each plane at 20 seconds:
  Flight NAL63 **15 feet** Flight WAL27 **10 feet**

  At 20 seconds, are the two planes in the same position on the jet route? Why or why not? **No. Flight NAL63 is at 15 feet, but Flight WAL27 is only at 10 feet.**

At 30 seconds:

- □ Find the distance traveled by each plane at 30 seconds:
  Flight NAL63 **7.5 feet** Flight WAL27 **15 feet**

- □ Find the new position of each plane at 30 seconds:
  Flight NAL63 **17.5 feet** Flight WAL27 **15 feet**

At 30 seconds, are the two planes in the same position on the jet route? Why or why not? **No. Flight NAL63 is at 17.5 feet, but Flight WAL27 is only at 15 feet.**

Activity 7.3F (cont.)

At 40 seconds:

- ◻ Find the distance traveled by each plane at 40 seconds:
  Flight NAL63 **10 feet** Flight WAL27 **20 feet**

- ◻ Find the new position of each plane at 40 seconds:
  Flight NAL63 **20 feet** Flight WAL27 **20 feet**

- ◻ At 40 seconds, are the two planes in the same position on the jet route? Why or why not? **Yes, each plane is at 20 feet. The planes are in conflict.**

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? **40 seconds**

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?
   Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.
   The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?
   The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

Activity 7.3F (cont.)

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?

   The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

Activity 7.3G—Graph Two Linear Equations:

The airplanes will conflict at 40 seconds, the time that corresponds to the point where the two lines intersect. At 40 seconds, each plane is 20 feet from the beginning of the jet route. Since the planes are traveling on the same route, the planes intersect at the point where their graphs intersect.

For the line corresponding to Flight WAL27, the slope of the line is 0.5 feet/sec, the speed of Flight WAL27.
For the line corresponding to Flight NAL63, the slope of the line is 0.25 feet/sec, the speed of Flight NAL63.

The vertical intercept of each line (0 and 10, respectively) corresponds to each plane's initial distance (0 feet and 10 feet, respectively) from the start of the jet route. In particular, 10 represents the 10-foot headstart of Flight NAL63.

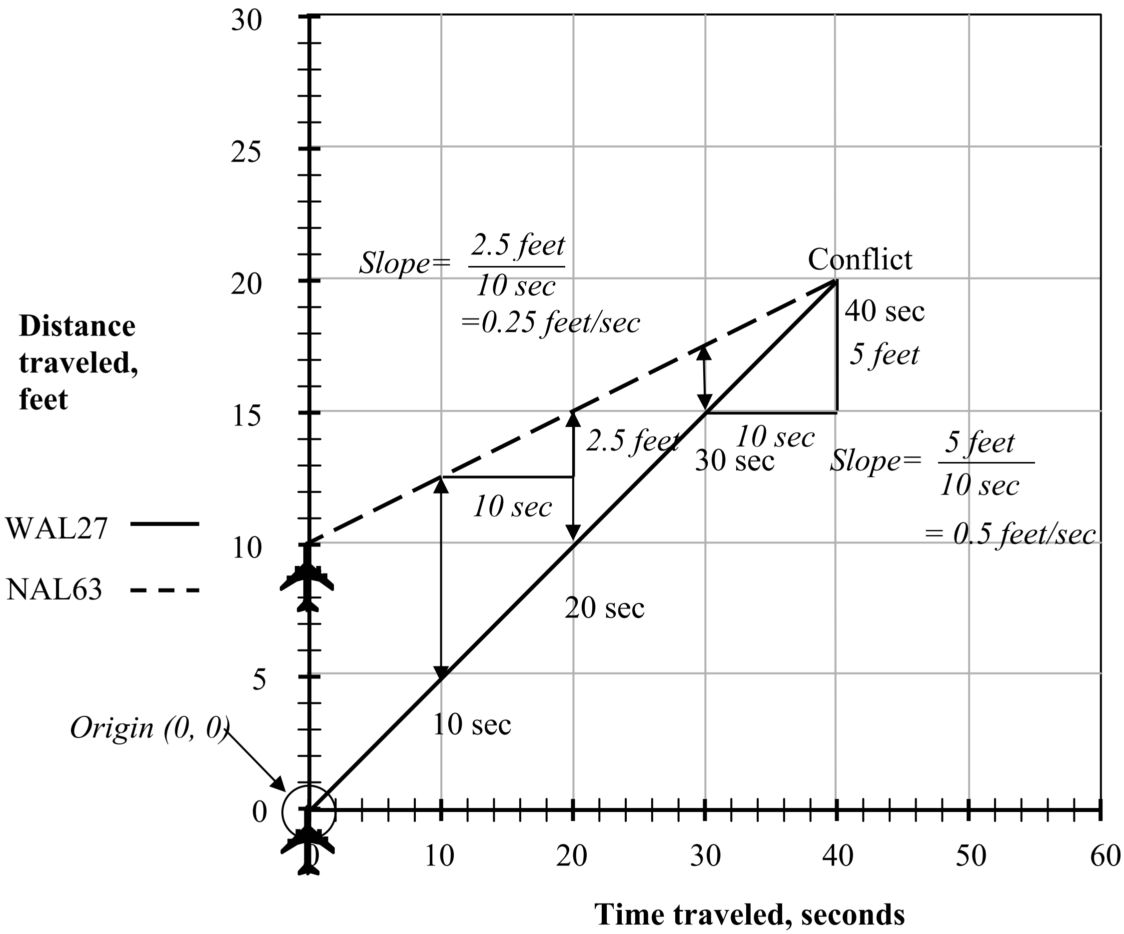
WAL27 $y = (1/2)x$

x	y
0	**0**
10	**5**
20	**10**
30	**15**
40	**20**

(10 ft headstart)
NAL63 $y = (1/4)x + 10$

x	y
0	**10**
10	**12.5**
20	**15**
30	**17.5**
40	**20**

Activity 7.3G (cont.)



1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63? That is, after how many seconds will each plane be at the same place at the same time? __**40 seconds**__

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? __**1/4 foot per second**__

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second?
   Flight WAL27 closes the gap at a rate of 1/4 foot per second.

4. Compare your answer to Question 3 with your answer to Question 2.
   The answers are the same. Flight WAL27 closes the gap at a rate of 1/4 foot per second, the difference in the plane speeds.

5. The speed of Flight WAL27 is **1/2** foot per second. Now suppose the speed of Flight WAL27 were **3/4** foot per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap between the planes?
   The new difference in the plane speeds is 1/2 foot per second. So Flight WAL27 would close the gap at a rate of 1/2 foot per second.

6. Compare your answer to Question 5 with your answer to Question 3. With the new faster speed, how many seconds will it take to close the gap?
   The new difference in speeds is twice the difference of the original speeds. So the new closing rate is twice the original closing rate. It will take 20 seconds to close the gap.

7. Write the number that is the slope of the solid line representing Flight WAL27.
   1/2 foot/second

8. Write the number that is the slope of the dotted line representing Flight NAL63.
   1/4 foot/second

9. What information does the slope of each line tell you about each plane?
   For Flight WAL27, the slope of the line is 1/2 foot/second, the speed of Flight WAL27. For Flight NAL63, the slope of the line is 1/4 foot/second, the speed of Flight NAL63.

Activity 7.4—After the Experiment and Activity 7.5—Posttest:

General Problem:

Two planes are traveling at different speeds on the same route.
The trailing plane is traveling faster than the leading plane.

This is not enough information to determine how long it will take the trailing plane to catch up to the leading plane because neither the plane speeds nor the starting separation distance is known.

However, students now know that the trailing plane will close the gap at a rate equal to the difference in the speeds of the planes. They can use this information to address the next two general scenarios:

- Suppose the difference in speeds is twice as great and the starting distance between the planes remains the same as the original starting separation distance. Then the amount of time for the trailing plane to catch up to the leading plane will be half as great.

  This is because the trailing plane closes the gap at a rate equal to the difference in the speeds of the planes. If that difference is twice as great, then the trailing plane is closing the gap at twice the original rate. So it will take half as much time to close the gap.

- Finally, suppose that the planes each travel at their original speeds, but the starting distance between the planes is twice as great. Then the amount of time will double for the trailing plane to catch up to the leading plane.

  This is because the trailing plane closes the gap at a rate equal to the difference in the speeds of the planes. If the gap is twice as great, then the trailing plane must close twice the distance at the original rate. So it will take twice as much time to close the gap.